Elements of a statistical test p462
- Null hypothesis
- Alternative hypothesis
- Test Statistic
- Rejection region

Rejection Region p462
The rejection region (RR) specifies the values of the test statistic for which the null hypothesis is to be rejected in favor of the alternative hypothesis.

Definition 10.1 p463
A type I error is made if $H_0$ is rejected when $H_0$ is true. The probability of a type I error is denoted by $\alpha$. The value of $\alpha$ called the level of the test.

A type II error is made if $H_0$ is accepted when $H_a$ is true. The probability of a type II error is denoted by $\beta$. P463
Ex 10.2 p466 An experimenter has prepared a drug dose level that she claims will induce sleep for 80% of people suffering from insomnia. In an attempt to disprove her claim, we administer her prescribed dosage to 20 insomniacs, and we observe $Y$ the number for which the drug dose induces sleep. We wish to test the hypothesis $H_0: p = 0.8$ versus the alternative $H_a: p < 0.8$.

Assume that the rejection region $\{y \leq 12\}$ is used.

a) In this problem, what is a type I error?
b) Find $\alpha$.
c) In this problem, what is a type II error?
d) Find $\beta$ when $p = 0.6$
e) Find $\beta$ when $p = 0.4$

10.3 Refer to Ex 10.2

a) Find the rejection region of the form $\{y \leq c\}$ so that $\alpha \approx 0.01$.
b) For rejection region in (a), find $\beta$ when $p = 0.6$

c) For rejection region in (a), find $\beta$ when $p = 0.4$
Definition 10.3 (Power of a test) p508

Suppose $W$ is a test statistics and RR is the rejection region for a test of a hypothesis involving the value of a parameter $\theta$. Then the power of the test, denoted by $\text{power}(\theta)$, is the probability that the test will lead to rejection of $H_0$ when the actual parameter value is $\theta$.

Ex 10.76

Refer to Ex 10.2. Find the power of the test for the following alternatives.

a) $p = 0.4$

b) $p = 0.5$

c) $p = 0.6$

d) $p = 0.7$

e) Sketch the graph of the power function.

Note: $\text{Power}(\theta) = 1 - \beta(\theta)$
**Definition 10.4** p509

If a random sample is taken from a distribution with parameter $\theta$, a hypothesis is said to be a *simple hypothesis* if that hypothesis uniquely specifies the distribution of the population from which the sample is taken. Any hypothesis that is not a simple hypothesis is called a *composite hypothesis*.

**Theorem 10.1** p510

**The Neyman-Pearson Lemma**

Suppose that we wish to test the simple null hypothesis $H_0 : \theta = \theta_0$ versus the simple alternative hypothesis $H_a : \theta = \theta_a$, based on a random sample $Y_1, Y_2, \ldots, Y_n$ from a distribution with parameter $\theta$. Let $L(\theta)$ denote the likelihood of the sample when the value of the parameter is $\theta$. Then, for a given $\alpha$, the test that maximizes the power at $\theta_a$ has rejection region $R$, determined by

$$L(\theta_0) < k.$$  

The value of $k$ is chosen so that the test has desired value for $\alpha$. Such a test is a most powerful $\alpha$-level test for $H_0$ versus $H_a$. 

Example 10.22 p510
Suppose that $Y$ represents a single observation from the p. d. f given by

$$f(y) = \begin{cases} \theta y^{\theta - 1}, & 0 < y < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find the most powerful test with significance level $\alpha = 0.05$ to test $H_0 : \theta = 2$ versus the simple alternative hypothesis $H_a : \theta = 1$. 
Example 10.23 p511
Suppose that $Y_1, Y_2, \ldots, Y_n$ constitute a random sample from a normal distribution with unknown mean $\mu$ and known variance $\sigma^2$. We wish to test $H_0 : \mu = \mu_0$ against $H_a : \mu > \mu_0$. Find the uniformly most powerful test with significance level $\alpha$. 