Theorem 9.5 Rao-Blackwell Theorem

Let \( \hat{\theta} \) be an unbiased estimator for \( \theta \) such that \( V(\hat{\theta}) < \infty \). If \( U \) is a sufficient statistic for \( \theta \), define \( \hat{\theta}^* = E(\hat{\theta} | U) \). Then for all \( \theta \),

\[
E\left(\hat{\theta}^*\right) = \theta \quad \text{and} \quad V(\hat{\theta}^*) < V(\hat{\theta}).
\]
Example 9.6 p437
Let $Y_1, Y_2, \ldots, Y_n$ denote a r. s. from a Bernoulli($p$) distribution. Use the factorization to find a sufficient statistic that best summarizes the data. Give a minimum variance unbiased estimator (MVUE) for $p$. 

Example 9.7 p437
Let \( Y_1, Y_2, \ldots, Y_n \) denote a r. s. from a Weibull density given by
\[
f(y) = \begin{cases} 
\frac{2y}{\theta} e^{-y^2/\theta}, & y > 0 \\
0, & \text{elsewhere}.
\end{cases}
\]
Find an MVUE for \( \theta \).
The Method of moments 9.6 p444

$k$th moment of $Y$ about the origin, $\mu_k' = E(Y^k)$

$k$th sample moment $m_k' = \frac{1}{n} \sum_{i=1}^{n} Y_i^k$.

Method of moments p444

Choose as estimates those values of the parameters that are solutions of the equations $\mu_k' = m_k'$, for $k = 1, 2, \ldots, t$ where $t$ is the number of parameters to be estimated.

Example 9.11 p444 A random sample of $n$ observations, $Y_1, Y_2, \ldots, Y_n$ is selected from $\text{Unif}(0, \theta)$ where $\theta$ is unknown. Use the method of moments to estimate $\theta$. Is this estimator consistent?
Ex 9.62 p447
Suppose that $Y_1, Y_2, \ldots, Y_n$ is a random sample from a poisson distribution with mean $\lambda$.
Find the method of moments estimator of $\lambda$.

Ex 9.64 p447
If $Y_1, Y_2, \ldots, Y_n$ is a random sample from a normal distribution with mean $\mu$ and variance $\sigma^2$, find the method of moments estimators of $\mu$ and $\sigma^2$. 
The method of Maximum likelihood 9.7 p448

- Suppose that the likelihood function depends on $k$ parameters $\theta_1, \theta_2 \ldots \theta_k$. Choose as estimates those values of the parameters that maximize the likelihood $L(y_1, y_2, \ldots, y_n | \theta_1, \theta_2 \ldots \theta_k)$.

Example
Suppose $Y_1, Y_2, \ldots, Y_n \sim \exp(\theta)$. Determine the maximum likelihood estimator (MLE) of $\theta$. 
Example 9.16 p451
Let $Y_1, Y_2, \ldots, Y_n$ be a random sample from a normal distribution with mean $\mu$ and variance $\sigma^2$. Find the MLEs estimators of $\mu$ and $\sigma^2$. 
Invariance property of MLEs p452

- If \( t(\theta) \) is a one-to-one function of \( \theta \) and \( \hat{\theta} \) is the MLE of \( \theta \), then the MLE of \( t(\theta) \) is given by \( t(\hat{\theta}) \).

Ex 9.72 p453 Suppose that \( Y_1, Y_2, \ldots, Y_n \) is a random sample from a poisson distribution with mean \( \lambda \).

a) Find the MLE (\( \hat{\lambda} \)) of \( \lambda \).

b) Find the expected value and variance of \( \hat{\lambda} \).

c) Show that \( \hat{\lambda} \) is consistent for \( \lambda \).

d) What is the MLE for \( P(Y = 0) = e^{-\lambda} \).

Suggested problems for the week of Mar 21