Word of the day (October 20, 2006)

Persistence

Persistence is the twin sister of excellence. One is a matter of quality; the other, a matter of time.

Permanence, perseverance and persistence in spite of all obstacles, discouragement, and impossibilities: It is this, that in all things distinguishes the strong soul from the weak

Example: 4.31 (page 315) poker examples --- Poker: 52 cards, 13 diamonds, 13 spades, 13 hearts, 13 clubs, 4 Aces

1) At the beginning what is the chance a poker player get an ace? → 4/52=1/13

2) If he noticed that he has had 4 cards in hand, and one is ace, *then given what he has had*, what is the probability that he got a second ace?

P(ace | 1 ace in 4 visible cards) = 3/48

Conditional Probability p315

- P (B | A) is a conditional probability, that is, it gives the probability of one event (e.g. the next card is an ace) under the condition that we know another event (exactly, 1 of 4 visible cards is an ace)
- When $P(A) > 0$, the conditional probability of B given A is

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

- We can rewrite it, see the diagram for the interpretation (shall draw manually)

Example: poker example again 4.33 (page 317)
-- what to have two diamonds in a row, he has had 11 cards, 4 are diamonds. What is the probability of getting two diamonds in a row?

P (first card diamond) = 9/41
P (second card is diamond | fist one is diamond) = 8/40
→ P(first, second both diamond) = P (first card diamond) P (second card is diamond | fist one is diamond)
= 9/41 * 8/40 = 0.044 → need good luck ! ☺

Bayes rules:

If A and B are any events whose probabilities are not 0 or 1,

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A^c)P(A^c)}$$

In fact, the above formula is just another expression of the conditional probability:

The numerator = P(B and A) ;

The denominator = P (B and A) + P (B and A^C) = P(B)

Independent events p322

Two events A and B are both have positive probability are **independent** if

$$P(B|A) = P(B).$$

The tree diagram –to illustrate how to solve problems that have several stages

Ex: The fraction of persons in a population who have a certain disease is 0.01. A diagnostic test is available to test for the disease. But for a healthy person the chance of being falsely diagnosed as having the disease is 0.05, while for someone with the disease the chance of being falsely diagnosed as healthy is 0.2.Suppose the test is performed on a person selected at random from the population.

Tree diagram here:

a) What is the probability that the test shows a positive result?

P(positive)=P(healthy and test positive) + P(disease and test positive)= 0.01*0.8 + 0.99*0.05

b) What is the probability that the person selected at random is one who has the disease but is diagnosed healthy?

0.2 * 0.01

c) What is the probability that the person is correctly diagnosed and is healthy?

0.99*0.95

d) If the test shows a positive result, what is the probability this person actually has the disease?

A= positive result; B= the person has the disease

P(B | A)= P(A and B) / P(A) = (0.01*0.8)/ result of (a)

e) If the test shows a positive result, what is the probability this person actually healthy?

f) If the test shows a negative result, what is the probability this person actually has the disease?

g) If the test shows a negative result, what is the probability this person actually healthy?

Note 1: questions e), f) and g) are similar as d).

Note 2: Tree diagrams combine the *addition and multiplication* rules.

- The *multiplication* rule says that the probability of reaching the end of any complete branch is the product of the probabilities written on its segments.
- The probability of any outcome is then found by *adding* the probability eof all branches that are part of the events.
- Multiplication rule: P(A and B)=P(A)P(B) if A and B are independent
- Addition rule: P(A or B) =P(A) +P(B) if A and B are disjoint

Ex: The distribution of blood types among white Americans is approximately as follows: 37% type A, 13% type B, 44% type O, and 6% type AB. Suppose that the blood types of married couples are independent and that both the husband and wife follow this distribution.

(a) An individual with type B blood can safely receive transfusions only from persons with type B or type O blood. What is the probability that the husband of a woman with type B blood is an acceptable blood donor for her? 13% + 44%

(b) What is the probability that in a randomly chosen couple the wife has type B blood and the husband has type A? (13% * 37%)/2

(c) What is the probability that one of a randomly chosen couple has type A blood and the other has type B?

13% * 37% --- note: pay attention to the difference of b) and c)

(d) What is the probability that at least one of a randomly chosen couple has type O blood?

$1- (1-0.44)^2$

Question 13 Term Test Summer 99

A space vehicle has 3 ‘o-rings’ which are located at various field joint locations. Under current whether conditions, the probability of failure of an individual o-ring is 0.04.

a)A disaster occurs if any of the o-rings should fail. Find the probability of a disaster. State any assumptions you are making.

P(disaster) = 1- P(all work) = 1-0.96*.96*.96 = 0.115

Assuming independent failures.

b) Find the probability that exactly one o-ring will fail.

P(FWW) = 0.04*.96*0.96 = 0.037 = P(WFW) = P(WWF)

P(exactly one) = 0.037 + 0.037 + 0.037 = 0.111

Question 23 Final exam Dec 98
A large shipment of items is accepted by a quality checker only if a random sample of 8 items contains no defective ones. Suppose that in fact 5% of all items produced by this machine are defective. Find the probability that the next two shipments will both be rejected.

$(1-0.95^8)^2 = 0.1133$

Ex: An automobile insurance company classifies drivers as class A (good risks), class B (medium risks), and class C (poor risks). Class A risks constitute 30% of the drivers who apply to them for insurance, and the probability that such a driver will have one or more accidents in any 12-month period is 0.01. The corresponding figures for class B are 50% and 0.03, while those for class C are 20% and 0.10.

The company sells Mr. Jones an insurance policy, and within 12 months he had an accident. What is the probability that he is a class A risk?

Ex (Q9 Final exam Dec 2001)
You are going to travel Montreal, Ottawa, Halifax, and Calgary, but the order is arbitrary. You put 4 marbles in a box, each one labeled for one city, and draw randomly. The first marble is the first city you will visit, the 2nd marble indicates your 2nd stop etc.
What is the probability that you visit Ottawa just before or just after you visit Montreal?
Ans 0.5