Linear Combination of Two Random Variables

• Let $X_1$ and $X_2$ be random variables with

$$E(X_1) = \mu_1, \quad E(X_2) = \mu_2, \quad \text{var}(X_1) = \sigma_1^2, \quad \text{var}(X_2) = \sigma_2^2 \quad \text{and} \quad \text{Cov}(X_1, X_2) = \sigma_{12}$$

• Let $Y = aX_1 + bX_2$. Then,…
Linear Combination of Independent R.V

- Let $X_1, X_2, \ldots, X_n$ be independent random variables with
  \[ E(X_i) = \mu_i \quad \text{and} \quad \text{var}(X_i) = \sigma_i^2 \]

- Let $Y = a_1X_1 + a_2X_2 + \cdots + a_nX_n$. Then,…
Linear Combination of Normal R.V

• Let $X_1, X_2, \ldots, X_n$ be independent normally distributed random variables with

$$E(X_i) = \mu_i \quad \text{and} \quad \text{var}(X_i) = \sigma_i^2$$

• Let $Y = a_1X_1 + a_2X_2 + \cdots + a_nX_n$. Then,…
Examples
The Chi Square distribution

- If $Z \sim N(0,1)$ then, $X = Z^2$ has a Chi-Square distribution with parameter 1, i.e., $X \sim \chi^2(1)$.

- Can proof this using change of variable theorem for univariate random variables.

- The moment generating function of $X$ is

$$m_X(t) = \left( \frac{1}{1 - 2t} \right)^{1/2}$$
Important Facts

• If $X_1 \sim \chi^2(v_1), \ X_2 \sim \chi^2(v_2), \ldots, \ X_k \sim \chi^2(v_k)$, all independent then,

$$T = \sum_{i=1}^{k} X_i \sim \chi^2_{\sum_{i}^{k} v_i}$$

• If $Z_1, Z_2, \ldots Z_n$ are i.i.d N(0,1) random variables. Then,

$$Y = X_1^2 + X_2^2 + \cdots + X_n^2 \sim \chi^2(n)$$
Claim

- Suppose $X_1, X_2, \ldots, X_n$ are i.i.d normal random variables with mean $\mu$ and variance $\sigma^2$. Then, $Z_i = \frac{X_i - \mu}{\sigma}$ are independent standard normal variables, where $i = 1, 2, \ldots, n$ and

$$\sum_{i=1}^{n} Z_i = \sum_{i=1}^{n} \left( \frac{X_i - \mu}{\sigma} \right)^2 \sim \chi^2(n)$$
Important facts

• Suppose $X_1, X_2, \ldots X_n$ are i.i.d normal random variables with mean $\mu$ and variance $\sigma^2$. Then,

$$\frac{1}{\sigma^2} \sum_{i=1}^{n} (X_i - \bar{X})^2 \sim \chi^2_{(n-1)}$$

Proof:…

• This implies that...

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{(n-1)}$$

• Further, it can be shown that $\bar{X}$ and $s^2$ are independent.