Suggested Work # 5

Read the lecture which was posted. This included the in-class lecture.

Problems

1. Let $X_1, \ldots, X_m$ be iid $N(\mu, \sigma^2)$. Set $\bar{X} = \sum_{k=1}^{m} X_k$ and $s^2 = \frac{1}{m-1} \sum_{k=1}^{m} (X_k - \bar{X})^2$. Show $\frac{\bar{X} - \mu}{s/\sqrt{m}} \sim t(m-1)$.

2. Let $X_1 \sim \text{gamma}(r_1, \lambda)$ be independent of $X_2 \sim \text{gamma}(r_2, \lambda)$. Obtain the pdf of $W = \frac{X_1}{X_1 + X_2}$ and use this to obtain the pdf of $Y = \frac{U/n}{V/m}$, where $U \sim \chi^2(n)$ is ind of $V \sim \chi^2(m)$.

3. Let $\underset{2\times 2}{\Sigma}$ be a variance matrix which is invertible and $\underset{2\times 1}{\underset{\sim}{\mu}}$ a vector of constants. Write $\Sigma = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_2\sigma_1 & \sigma_2^2 \end{pmatrix}$. Let $\underset{\sim}{Z} = \begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix}$ have iid $N(0,1)$ components. (a) Find a lower Triangular matrix T so that

$$\underset{\sim}{Y} = \underset{\sim}{\mu} + T \underset{\sim}{Z} \sim N(\underset{\sim}{\mu}, \Sigma)$$

(b) Obtain the conditional pdf $f(y_2 | y_1) = \frac{f(y_1, y_2)}{f_{Y_1}(y_1)}$ & use it to calculate

$$E(Y_2 | Y_1 = y_1) = \int_{-\infty}^{\infty} y_2 \, f(y_2 | y_1) \, dy_2$$

(c) Obtain $E(Y_2 | Y_1 = y_1)$ from $\underset{\sim}{Y} = \underset{\sim}{\mu} + T \underset{\sim}{Z}$.

4. Let $\{N(t) \mid t \geq 0\}$ have stationary independent increments and suppose $P(N((t, t+h]) = 1) = \lambda(t) h + o(h)$ while $P(N((t, t+h]) > 1) = o(h)$. Here $\lambda(t) > 0$, $N(A) = \#$ of pts in A and $N(t) = N([0,t])$. Assume $N(0) = 0$. Show $N(t) \sim \text{Poisson}(m(t))$, where $m(t) = \int_0^t \lambda(u) \, du$.