Behrens-Fisher

Recall \( N \mu_1 \sigma_1^2 \)

\[
f = \frac{(2\pi)^{\frac{d}{2}}}{\sqrt{\det(S)}} \exp\left\{ -\frac{1}{2} \sum_{i=1}^{n} (y_{i} - \mu)^2 \right\}
\]

- Bayes & freq
- Why two?
- Accuracy
- Scalar
- Vector
- One Normal?
- Two Normals?
- General?

Behrens 1929, Fisher 1935

- Neyman 1937
- David Stone Zadek 1973
- Welch 1947
- Linde 1958
- Welch-Pearson 1963
- Gosh & Kim 2001
- Emgine 2009

A Behrens Fisher 2 samples N

\[ y_{i} - \mu = t \frac{\sigma}{\sqrt{n}} \]

Conf. Interval \( 95\% \)

Invent "Sampling distribution of \( t \)" \( 95\% \) interval.

Two approaches
- Invent Sample Stats
- Invent Population Stats

Why? What? How?
Behrens–Fisher
1929 1935
Neyman 1937
⇒ Lecture to Fisher

Fine Tuning Confidence Structure of Confidence

β
Neyman (1937)
Gave it name

1. For each θ, choose A(θ) on S with

2. On R x S form A = {θ} x A(θ)

3. Any y, calc C(y) = \{θ: y ∈ A(θ)\}

= All θ values accepted by A(θ)

y ∈ A(θ) ⇔ (y, θ) ∈ A ⇔ θ ∈ C(y)

\[ I_{A}(y, θ) \sim \text{Bern}(β) \times \text{Pivotal} \]

Sorting out: Confidence distn → α on S x D

Distn to "θ" from M + Data

of Bayes too!
D Welch 1947

Behrens Fisher

\[ t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \]

Approx. denom. by \( X \)

Targeted on B- F

Not student approx N o 1

No general soln

Pragmatic

Works well

Use 2 moments get df
Behrens-Fisher 1929 1935
Scalar Exp. model

\[ f = \exp \{ s(y) \varphi(\theta) - K(\theta) \} h(y) \quad \text{on } \mathbb{R}^d \]
\[ \theta \in \mathbb{R} \]

Use of prior \( \rightarrow \) Conf Intervals

\[ i(\theta) = E\{ - \varphi \varphi^t \} \]
\[ d\beta(\theta) \overset{1/2}{=} (\theta) d\theta \]
\[ \beta = \int d\beta(\theta) = \int \theta^{1/2} (\theta) d\theta \quad \text{Reparametrize } \beta \]

"Location"

Prior \( \overset{i^{-1/2}}{\sim} \)

Jeffreys Asymptotic

\[ \hat{\beta}(y) - \beta(\theta) = \text{fixed dist} \]

Power

freq Conf \( \leftrightarrow \) Bayes Calc

\[ \Rightarrow \text{Use "root info as prior} \]

Get Conf = posterior

2nd order

N-P Precursor to ITOL
Behrens - Fisher  \( N \) 2-sampler  \( S = \mu_1 - \mu_2 \)

Ghosh & Kim

\[
\sqrt{\frac{S^2}{n_1 \sigma_1^2} + \frac{S^2}{n_2 \sigma_2^2}} \sim \chi^2(\nu_1 + \nu_2)
\]

1st  \( \frac{d\mu_1}{d\sigma_1^2} \)

2nd  \( \frac{d\mu_2}{d\sigma_2^2} \)

Jeffrey x Jeffreys

Results from Simulations

Behrens - Fisher - Obvious (Jeff) \( \Rightarrow \) Ghosh & Kim 2001

Frequentist

\[
\Phi\left(\frac{S}{\sqrt{n_1 + n_2}}\right)
\]

\( S = \mu_1 - \mu_2 \)
Always have

\[ z = \text{Dan}(w) \]

\[ \text{Desurance} \]

\[ 2 \left( (e-9) \right) \]

\[ \frac{2}{3} \]

Due in 2 hrs

\[ t = 1 \]
2 Normal Samples

One Sample

\[ Y_i - \mu \sim N(\mu, \sigma^2) \]

\[ \hat{\mu} = \frac{1}{n} \sum Y_i \]

\[ \sqrt{\frac{\sum (Y_i - \hat{\mu})^2}{n}} \]

\[ r = r(w) = \frac{\text{sqr}(\hat{\mu} - \mu)}{\sqrt{2(\hat{\mu} - \hat{\mu})^2}} \]

\[ \chi^2 = \sum (x - \bar{x})^2 \]

\[ \chi^2 \sim \chi^2(n-2) \]

Where do these come from?

- Substitution
- Picture here: \[ N \times \mu \]

Highly Accurate: \[ \text{Stu}(n-1) \]

1DOL

Here \( M \sim \chi^2 \)

\[ t \sim \chi^2(n-2) \]

\[ \log \text{Likelihood} \sim \frac{x^2}{2} \]

\[ N \approx \log t \]
\[ NmG^2 \begin{vmatrix} y_1 & \cdots & y_n \end{vmatrix} \]

\[ S_{LR} = r_r = \log (b - y) \sqrt{2(b - l_0)} = \text{ord to get available generally M, y} \]

Calculate

\[ \text{MLE depastive} \]

\[ \text{Can}\ \text{par} \ \phi = \left( \frac{1}{\phi}, \frac{1}{\phi^2} \right) \]

\[ Nm \text{ std par} \]

\[ \text{Fixed phi} \]

\[ \text{Constrained param space} \]

\[ \text{Intersect in y} \]

\[ \text{Fix} \mu \]

\[ \text{Parameter space} \]

\[ \left( \frac{r}{\phi}, \frac{\mu}{\phi^2} \right) \]
Clairification Next time ?

\[ \Phi = 1 \hat{\mathbf{y}} - \hat{\mathbf{X}} \mu \]

Tested \( \Phi \) on new \( \mathbf{X} \) space

\[ \hat{\mathbf{y}} - \hat{\mathbf{X}} \mu \]

\[ N \mu \sigma^2 \]

\[ \mu \sigma^2 \rightarrow \Phi_1, \Phi_2 \]

\[ \mu_1, \mu_2, \sigma_1^2, \sigma_2^2 \]

Can 3 general par \( \beta \) para

\[ \phi(x-n', \cos q') \]

General Saracce model Simulation B-F

\[ \theta = 1 \hat{\mathbf{y}} - \hat{\mathbf{X}} \mu \]

\[ \text{(Info Corr)} \]

Reprodus

\[ \phi(x-n', \cos q') \]

\[ \text{Student-T} \]
Scalar $\theta^{*}(y; \theta)$

Simulations: $-\text{natural}$

$f(\theta) = \log f(y^*; \theta) \quad L(\theta)$

$\mathbb{E} \frac{1}{n} S_{LB}$

MLE distribution

$\theta \sim \mathcal{N}(n^{-1} \text{log} q_{\hat{\theta}})$

Asymptotics: Reproduce to

Same generally (same)

Separates nuisance

$\exp \left\{ L(\theta) + \frac{1}{n} q(\hat{\theta}) \right\} \text{HLS}$

$N \approx 18 \sqrt{n}$

Exp $3 \sqrt{n}$

$\Rightarrow 3 \text{rd}$

Just Exp