How does $\Theta$ affect model?

Statistical Model
Data

Change $\Theta$ to $\Theta + d\Theta$
Probability is moved!

Contiuity: use it

$\text{Peter McCullagh Parameter?}$
How does $\Theta$ affect model?

Statistical Model

Data

Change $\Theta$ to $\Theta + d\Theta$

Probability is moved!

Distribution shifts; How much?

Stochastically inc. say
How does $\Theta$ affect model?

Statistical Model
Data

Change $\Theta$ to $\Theta + d\Theta$
Probability is moved!

Distribution shifts; How much?

$p = F(y; \Theta)$

Find $y + dy$ having prob $p$ to left!

$p = F(y + dy; \Theta + d\Theta) \rightarrow$

Solve for effect "dy"

What happens to that "unit mass of stuff"?
How does $\Theta$ affect model?

Statistical Model
Data

Change $\Theta$ to $\Theta + d\Theta$
Probability is moved!

Distribution shifts; How much?

$$p = F(y; \Theta)$$

Find $y + dy$, having prob $p$ to left!

$$p = F(y + dy; \Theta + d\Theta)$$

Solve for effect "$dy$"

Take total differential:

$$\theta = \frac{\partial F}{\partial y} dy + \frac{\partial F}{\partial \Theta} d\Theta = F_y(y; \Theta) dy + F_\Theta(y; \Theta) d\Theta$$
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Take total differential:

$$\theta = \frac{\partial F}{\partial y} dy + \frac{\partial F}{\partial \Theta} d\Theta = F_y(y; \Theta) dy + F_{\Theta}(y; \Theta) d\Theta$$

Shift caused by $\Theta \rightarrow \Theta + d\Theta$

$$dy = -\frac{F_{\Theta}(y; \Theta)}{F_y(x; \Theta)} d\Theta$$
Another "Take"

use quantile function

\[ p = F(y; \Theta) \]

Solve for \( y \) as function of \( \Theta \) \& \( p \)

Let's look the other way!
Another "Take"

Use quantile function

\[ p = F(y; \theta) \]

Solve for \( y \) as function of \( \theta \)? \( p \) or \( \theta \)? Let's look the other way!

\[ qf \quad y = F^{-1}(p; \theta) = y(p; \theta) \]
Another "Take"

Use quantile function

\[ p = F(y; \theta) \]

Solve for \( y \) as function of \( \theta \) & \( p \)

Let's look the other way!

\[ y = F^{-1}(p; \theta) = y(p; \theta) \]

Same "information"

Read in reverse order

\( p \leftrightarrow q \) or \( q \leftrightarrow p \)
Another "Take"

Use quantile function

\[ p = F(y; \theta) \]

Solve for \( y \) as function of \( \theta \) \( \Rightarrow p \)

Let's look the other way!

\[ q = F^{-1}(p; \theta) = y(p; \theta) \]

Same information

Read in reverse order

\[ p \leftrightarrow q \text{ or } q \leftrightarrow p \]

How does \( \theta \) affect \( y \)?
Another "Take"

use quantile function

\[ p = F(y; \theta) \]

Solve for \( y \) as function of \( \theta \)? \( p \)
Le look the other way!

\[ q = F^{-1}(p; \theta) = y(p; \theta) \]

\[ \text{Same information} \]
Read in reverse order
\( p \leftrightarrow q \) or \( q \leftrightarrow p \)

How does \( \theta \) affect \( y \)?

\[ \frac{dy}{d\theta} = \frac{\partial y(p; \theta)}{\partial \theta} \]

Easy: Differentiate quantile fn.
How does $\Theta$ affect model?

Statistical Model
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Change $\Theta$ to $\Theta + d\Theta$
Probability is moved!
How does $\Theta$ affect model?

Statistical Model

Data

Change $\Theta$ to $\Theta + d\Theta$

Probability is moved!

Slide 5

$dy = - \frac{F_{\Theta}(y; \Theta)}{F_y(x; \Theta)} d\Theta$

Slide 10

$dy = \frac{dy(p; \Theta)}{\partial \Theta} d\Theta$
How does $\Theta$ affect model?

Statistical Model

Data

Change $\Theta$ to $\Theta + d\Theta$

Probability is moved!

Slide 5

$$dy = - \frac{F_{\Theta}(y; \Theta)}{F_{\Theta}(x; \Theta)} d\Theta$$

Slide 10

$$dy = \frac{dy(p; \Theta)}{d\Theta} d\Theta$$

$F(y; \Theta)$ fixed = $p$ say

Differentiate via total derivative
How does $\theta$ affect model?

Statistical Model

Data

Change $\theta$ to $\theta + d\theta$

Probability is moved!

Slide 5

$$dy = -\frac{F_{y \theta}(y; \theta)}{F_y(x; \theta)} d\theta$$

Slide 10

$$dy = \frac{d y(p; \theta)}{d \theta} d\theta$$

$F(y; \theta)$ fixed = $p$ say

Differentiate via total derivative

For fixed $p$ write $y$ as fn of $\Theta$; then differentiate
How does $\Theta$ affect model?

Statistical Model

Data

Change $\Theta$ to $\Theta + d\Theta$

Probability is moved!

Slide 5

\[ dy = - \frac{\partial_y(y; \theta)}{F_y(x; \theta)} d\theta \]

Slide 10

\[ dy = \frac{d_y(p; \theta)}{d\theta} d\theta \]

Same result!

Two calculus roots to $n(\theta) = \frac{dy}{d\theta}$

How $\Theta$ moves data!

Velocity re $\Theta$
Works generally:

Model: $f(y; \theta)$

Data: $y^0$

dim $y = n$

dim $\theta = p$ \quad \Rightarrow \quad \text{Regularity Asymptotics Continuity} \quad \text{for } n \geq p
Works generally:
Model: \( f(y; \theta) \)
Data: \( y^0 \)

\( df \rightarrow gf \)
Use quantile fn

\( \dim y = n \)
\( \dim \theta = p \)

Regularity
Continuity

\( y = y(p; \theta) \)
\( \sim \sim \sim \sim \) Vector p value
Works generally:
Model: \( f(y; \theta) \)
Data: \( y_0 \)

1) df \( \rightarrow \) qf
Use quantile fn

2) Differentiate

\[
\frac{\partial y}{\partial \theta} = \left\{ \frac{\partial y_i}{\partial \theta_j} \right\} = V(\theta) = (v_1(\theta), \ldots, v_p(\theta)) \quad p \text{ col vectors in } \mathbb{R}^n
\]

\( V(\theta) \) is the Information Matrix

Regularity
Continuity
Change in \( \theta \) and its effect on model, on data point...

Indep coords Vectors \( \theta \)
Dep. coord \( \cdots \) Q's \( \cdots \) /8
Works generally:

Model: \( f(y; \theta) \)

Data: \( y^0 \)

\( \dim y = n \)

\( \dim \theta = p \)

Regularity

Continuity

1) \( df \rightarrow qf \)

Use quantile fn

2) Differentiate

\[ \frac{\partial y}{\partial \theta} = \left\{ \frac{\partial y_i}{\partial \theta} \right\} = V(\theta) = (v_1(\theta), \ldots, v_p(\theta)) \]

\[ n \times p \]

3) Get: \[ dy = V(\theta) d\theta \] what can you do?
Works generally:

Model: \( f(y; \theta) \)

Data: \( y^0 \)

1) \( df \to qf \)
   Use quantile fn

2) Differentiate
   \[ \frac{\partial y}{\partial \theta} = \begin{pmatrix} \frac{\partial y_1}{\partial \theta} \\ \vdots \\ \frac{\partial y_n}{\partial \theta} \end{pmatrix} = V(\theta) = (v_1(\theta), \ldots, v_p(\theta)) \]
   \[ n \times p \]

3) Get:
   \[ dy = V(\theta) d\theta \]
   \( \begin{cases} \text{Change } \hat{\theta} \to \hat{\theta} + d\theta \\ \text{Probability flows} \\ \text{Contours ancillary} \end{cases} \)

   a) At \( \theta = \hat{\theta} \)
   \[ dy = V(\hat{\theta}) d\theta = V d\theta \]
Works generally:

Model: $f(y; \theta)$

Data: $y^0$

$\dim y = n$

$\dim \theta = p$

Regularity

Continuity

1) $df \rightarrow qf$

Use quantile fn

2) Differentiate

$$\frac{\partial y_i}{\partial \theta_j} = \{ \frac{\partial y_i}{\partial \theta_j} \} = V(\theta) = (v_1(\theta), \ldots, v_p(\theta))$$

$n \times p$

3) Get:

$$dy = V(\theta) d\theta$$

a) At $\theta = \hat{\theta}^0$

$$dy = V(\hat{\theta}^0) d\theta = V d\theta$$

Probability flows

Contours and auxiliary

b) At $\theta$

$$|dy| = |V(\theta)| d\theta = \sqrt{V'(\theta)V(\theta)}^{1/2} d\theta$$

$V(\theta)$ default prior

for Bayes

2nd

a) FRM Y \rightarrow JRSSB 2010 631-654

b) FFS \rightarrow Bernoulli 2010 1208-1223
a) \[ f(y, \theta) \quad y^0 \quad y = y(\theta; \beta) \]
\[
\text{dy} = V(\hat{\theta}) \text{d}\theta = V \text{d}\theta
\]
at \( y^0 \)

There is a 2nd order Ancillary ty+ to \( L(V) \)

Easiest part is \( V \)

Regress \( y = X\beta + \epsilon \)

Quantize \( \epsilon \)

Drop if \( y \) is centered on \( m \) points \( L(X) \)

Do you need ancillary

Auxiliary

Usually \( N \)

Change \( \theta \) to \( \hat{\theta} \)

Prob 'moves'

1st Deriv A

Ancillary

(2nd order A)

Theory

Bunsell Paper

Diff geom; discover Flow

Frobenius

Vector fields

(Diff Q)

Frob to 2nd order

Ancillary
1) There is a P-dimensional version (conditional on data info) at data $y^0$.

2) For $H_0: \phi$ only need $\frac{\partial l(\theta; y)}{\partial \theta} \bigg|_{y^0} = \phi(\theta)$.

Thus: From $y^0$, get $V = \frac{\partial y}{\partial \theta} \big|_{Data}$; calc $\phi(\theta)$.

Act as if $f(y; \theta) = \text{exponential } \phi(\theta)$.

Data $\Delta = 0$

$$f = \frac{1}{(2\pi)^k} \exp \left\{ l(\theta) + \phi(\theta) \mathbb{S} \right\} \int_{\phi} \mathbf{e}^{-\frac{1}{2} \mathbf{S} \phi} d\phi^0 \Delta = 0$$
Ex

\[ u_i = X \beta + \varepsilon_i \]

\[ z_i \sim f_i(z_i) \] may Logistic

Options

- LS & Bootstrap
- MLE & Bootstrap
- Default \( \frac{d\beta}{d\varepsilon} \) & MCMC

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Ex: Expe Can Link GLM

\[ y = X\beta + \sigma z \]

\[ z_i \sim f_i(z_i) \] Ray Logistic

\[ \frac{dy}{d\theta} = \left(\frac{x_{i1} \cdots x_{ip}}{\sqrt{n}}\right) \]

\[ y^0 = X\beta + \sigma z \]

\[ z = \frac{y^0 - X\beta}{\sigma} = z(\theta) \]

\[ \theta^0 = \hat{\beta}^0 \]

\[ V(\theta^0) = V \] \( p \)-value

\[ V(\theta) \] default

\[ \text{Default} = V(\theta) \frac{d\beta d\sigma}{\sigma} = \frac{d\beta d\sigma}{\sigma} \] - Jeffreys (modified)

- Right indep.

General error \[ \text{Def} \frac{d\beta d\sigma}{\sigma} \]
What can it do? Theory Continuity
Initial model => Approx model ... "Easy" to analyze

\[ y^* = X\beta + \sigma z \]  BC
\[ y = \sqrt{X(\beta + \sigma^2)} \]  Explicit model

\[ \text{Parameter } \lambda ? \]

Default prior: What parameters OK / Bad

UMPumb \to \]  Issues
UMPsim \to \]
Q1: Box Cox
Q2: UMPS UMPU UMPS
Q3: p-vector
Q4 Discrete?

Can you go beyond $O(n^{-1/2})$? How general? How to get quantile?

- Davidson & Reid: Score
  - $\frac{d\mu}{\mu} \Rightarrow \frac{dEV}{d\theta}$

- Likelihood
  - Vector

Theory $3\times 3$: 2nd $\Rightarrow$ Expected model while strong

Taylor Series expansions

Welch Pears ( )
Q: Model Regularity => Why/How go to Explo?

Why Explo Target:
- Welch Peers (scalar \( \theta \)) \( \Rightarrow \) Use Jeffreys.
- \( \text{Explo model} \Rightarrow 3\text{rd order Conf.} \)

Explo model \( \Rightarrow 3\text{rd order} \)

At \( y_0 \) \( \Rightarrow \) p-value at \( y_0 \)

\[
\begin{array}{c|c|c|c|c}
\theta & y & y^{2/3} & y^{1/3} & y^{1/2} \\
\hline
0 & 1 & \cdot & \cdot & \cdot \\
1 & 0 & \cdot & \cdot & \cdot \\
-1 & 0 & \cdot & \cdot & \cdot \\
\hline
\end{array}
\]

\( \phi(\theta) \)

Expand at \( y_0 \) at \( \theta_0 \)

Rescale

\( F(y_0; \theta) = 0.5 \)

3rd