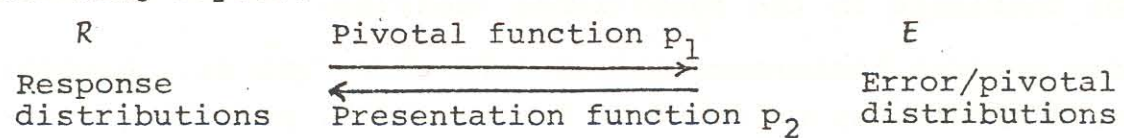


Comment on Barnard's Pivotal Inference  
D. Brenner and D.A.S. Fraser  
University of Toronto

Professor Barnard's paper proposes (1) a conditional inference method citing the location-scale model and mentioning the regression model, offers (2) support for the method in terms of pivotal functions and the ancillarity principle, mentions (3) the possibility of inference for error shape parameters.

1. The proposed conditional inference method has been available on the University of Toronto computer system since 1972, with a report on the computations and characteristics in Fraser (1975) and with further details on robustness and resistance properties in Fraser (1979, pp.30-46, 120-132, 206-250). In particular (ibid), computer simulations with graphical display showed how Cauchy conditional analysis worked well for samples that came in fact from near normal distributions whereas normal analysis was usually extremely bad for samples that came from low-student to Cauchy distributions. The conditional inference methods (estimates, tests, confidence intervals) were discussed (Fraser, 1979) for general models restricted only by closure properties needed to validate the conditioning step; also inference for shape parameters was examined with computer illustrations for location-scale, regression, directional, and bioassay models.

2. The support for the proposed conditional inference method is given by Professor Barnard in terms of pivotal functions and the ancillarity principle. The support is given by Fraser (1975, 1979) in terms of the error (or structural) model. Consider the following layout:



In the common cases where the response distributions  $R$  and error distributions  $E$  are on equivalent spaces, the pivotal function  $p_1$  is the inverse of the presentation function  $p_2$  and vice versa. Thus the pivotal model and the structural (and structured; Fraser, 1966, 1972, 1979) model are mathematically isomorphic and differs only in arguments used to support them.

The pivotal basis for the conditional inference method involves arbitrary steps (recourse to pivotal functions; choice of pivotal function) and the use of the ancillarity principle (self contradictions, Fraser 1979, pp.75-80). The structural basis involves objective support for the error distribution (Brenner and Fraser, 1980) and automatic conditioning based on observed error characteristics. The more general structured model (1972, 1979) reveals the lack of grounds for conditioning when the closure properties of the structural model do not obtain; this bears on the corresponding non-existence of a suitable ancillary in the pivotal version.

3. Inference for error shape parameters was examined in Fraser (1968, 1979) with computer analysis for common models in Fraser (1975, 1979); for example, the estimation of the degrees of freedom for the student family, and on the Weibull versus log-normal choice for lifetime data. The computer program used for these examples handles error families that can be given analytically as with the student family or hyperbolic family (Barndorff-Nielsen) or given numerically for direct computer input; thus, for example, the use of the student family was for convenience of illustration and not intrinsic to the conditional analysis.

4. Some general background comparisons of structural, pivotal, and mixed models may be found in Fraser (1979, pp.1-15).

References

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