STA 4412: Statistical Inference: New theory, techniques, current research, ...

Prerequisite: STA 422

Theme: Current theory, methods, research, conflicts ... usually to joint paper

Evaluation: Attendance, participation, contributions @ 1/3

Proposed

- Brief overview of STA 422
- Details, understanding, fine-tuning of

- What is statistical inference?
- Why does statistics have two approaches?
- New topics, research, directions

Statistics See Perception-role.pdf

Perception: 1) NSERC/grants/SSC 2) Science Feb 11, 2011 - Issue on Data
Science Sep 23, 2011 - Andrew Groves: Don't need clinical, again, and again, and again
Science Dec 2, 2011 - (replication but not from "core"

Two approaches + "Discipline" should be embarrassed!
Background: 422: 7 topics

1. p-value: Core of S.I. \( \rho(\gamma) = \text{Prob}\{\text{data} \leq 4\} \)

2. \( y(\theta) = \bar{y} \)

3. \( y = X\alpha + \epsilon \in N \)

4. \( \rho(\theta) = F(y; \gamma) = F_0(\gamma) \text{ Contri. (Uo, i)} \) Stat test \( y \) data

5. \( = S(y^0; \gamma) \) stochastic

6. Want in general \( \frac{\text{dim } y}{\text{dim } \theta} = p \)

7. \( \frac{\text{dim } \theta}{p} \geq n \)

8. \( \text{(Conf. from } \rho(\theta)) \)

9. \( \hat{\theta}_p \text{ is } \beta \)-level conf Reg. Reproducibly

10. \( \text{Upon conf Region:} \)

11. \( (\hat{\theta}_L, \hat{\theta}_U) \approx 85\% \text{ Conf Int} \)
(ii) \( \text{Obs } L(\theta) < L(\theta_0) \)\\
\( \text{Integrate with a distr. when } \theta \in \)\\
\( \text{1st + 2nd, 3rd} \)

(iii) \( \frac{L(\theta)}{L(\theta_0)} \approx N(\theta, \hat{\omega}(\theta)^{-1}) \) ; \( c(\theta) \to \)\\
\( c(\theta) = c(\theta_0) + \frac{1}{2} \hat{\omega}(\theta_0)^{-1} \theta - \theta_0^2 \) \( \approx \) \( c(\theta_0) + \frac{1}{2} \hat{\omega}(\theta_0)^{-1} (\theta - \theta_0)^2 \) \( \approx \) \( \) Quadratic \( \implies \) \( \hat{\theta} \) Distr. &\ etc \ etc \ 1st + order
Higher Order: Laplace \( O(n) \) sup expand. Terms are held by integrable regulars.

Key calculation:

\[
\int_0^\infty \phi(x) e^{-3x + 4x^2/2n} \, dx = 1 + (3a_4 + 5a_5) \frac{1}{2n}
\]

\( \phi(x) = e^{-3x + 4x^2/2n} \)

\[
\int\phi(x) \, dx = (2\pi)^{3/2} h(\lambda) \left[ \int_0^{1/2} e^{\lambda x} \, dx + O(n^{-2}) \right] \quad NB
\]

Saddlepoint: James 1934 in BN 1939 \( + \) \( + \) 

EM \( f(y; e) = \exp\{\phi(e) - k(01) h(y)\} \quad \frac{dy}{dx} = \text{pdf} \)

SP \( g = c \exp\{e - \hat{\theta}\} \cdot \frac{1}{\sqrt{2\pi}} \end{array} \begin{array}{l}
\int_{-\infty}^{\hat{\theta}} \text{(Gaussian ratio)} \quad \text{NB}
\end{array} \]

Scalar \( F(y; e) = e^{\phi(e) - \hat{\theta} y} \quad \text{for} \ y > 0 \quad \text{me dep} \)

Asymptotics: Regulations \( f(y; e) \)

\( f(e; y) = e^{\phi(e) - \hat{\theta} y} \left[ \int_0^{1/2} e^{\lambda y} \, d\lambda \right] \quad \text{const at data} \)

Just need target V \( \cdot \text{Re} \left[ e^{-i\theta} \cdot \int_0^{1/2} \right] \)

BN p-formula: Proposed BN 1980 BN invariance \( B \gamma = \hat{p} \quad \text{p dependence} \quad (\hat{p} = \frac{1}{\delta} \ln\lambda(y)) \)

Target \( \gamma_0 \) Too C V

Need \( V = (\mu - \mu_B) \), at data X e.
Taylor Series (As in Laplace)

"A Taylor series can be used in general models" (Ref: neumann.hec.ca/pages/jean-francois.plante/SSC2009/Poster.html)

Taylor expansion: Entry! Power! Use! Refer! Recommend.

209.pdf 

\[ \text{linear Taylor series} = \text{Taylor series} \]

\[ n, n', \text{etc.} \]
Bayes: Jeffreys' prior: Current version | Check!  Improve

Second order analysis (HOL; O(n^2)): Expect model: Easies

Scalar $y, \theta$ case $\sqrt{g(y)}$

$f(y; \theta) = \exp\left[ \phi(g(y) + K(\theta)) \right] h(y)$ on $\mathbb{R}^n$

$f(y; \theta) = \exp\left[ \phi - K(\theta) \right] h(y)$

Both monotonic in $\phi$ $Q_\phi = \phi h_n(\phi) (\phi^{-1} - 1)$

$f(y; \phi) = e^{\phi} \phi h_n \left( \phi \right) \frac{1}{\phi} \left( \phi^{-1} - 1 \right)$

$F(y; \phi) = \Phi \left( \phi - e^{-\phi} \right)$

$\Phi \left( \phi \right)$

$O(n^{-1/2})$
Exponential models... via log-model expansions

Forget!

Normal model .... Ø

Exotic model ... second order... easy/accurate... just need \{ \Phi(n-n'1g-\varphi/2) \} etc... easy.

\[
f(y; \theta) = c \exp \{ \phi(\theta) \Delta(y) + \kappa(\theta) \Psi(y) \} + \sum_{n=1}^{\infty} \frac{1}{n!} \exp \{ \Delta(y) \} \left( \psi(\theta) \right)^n
\]

Auto/para

\[
\lim_{n \to \infty} \frac{1}{\sqrt{n}} \sum_{j=1}^{n} \Theta_j = \{ \Theta \} \text{ sufficiently Corr.}
\]

Likelihood is correct at each point

"Fourier inversion... " Cheap!" Easy

"A Taylor view of \( \tau \) and \( \tau' \) in general models" #6 on web page... jean-francois

Exp Models (No X)

\[
f(y; \theta) \to f(y; \phi) \quad \text{as } n \to \infty
\]

\[
f(y; \theta) \to f(y; \phi) \quad \text{as } n \to \infty \quad \text{and} \quad \exp \left\{ \frac{\Delta(y)^2}{2} \right\} \text{ for large } \Delta(y)
\]

Regular

Asymptotic

\( n \to \infty \)
Case: dimo-daim - I Scalar

Euler Angle Integration: No Scale-up

Fig. 2.1: 222pm 2-2
Scalar Expt model: centered & scaled coordinates

\[ f(x; y) = \frac{1}{(2\pi)^{\frac{3}{2}}} \exp\left\{ -\frac{(x - \mu_x)^2}{2} - (y - \mu_y)^2 \right\} \left( 1 - \omega_3 \delta x^2 \right) \]

Does this integral to 1?

\[ \mathbb{E}\{a x^2 + b y^2 + c x^3 \mid y\} = 0 \quad \forall \phi \]

Centering, scaling: Regularly Arg: \(-\frac{1}{2} + s\phi\) Stable 

One parameter (math) \(\theta_3\) Get \(S^3\) etc as above

"Second new model" \(\Rightarrow 264\).pdf; Jean-Francois (and); simplified

\[ 264\).pdf -- why not go vector

Centering Scaling 2-3

\[ E\{e^2 + 3\delta\phi\psi\} = 0 \]

\[ N(\psi; 1) \]

NB 264.pdf
(2) Scalar $y$, Scalar $\theta$

- $f(y, \theta)$

- $y^\circ$

- "frequentist"
  - $p(\theta) = \frac{\text{Proportion} \text{ data on para scale } \theta}{\text{Proportion} \text{ left if data not on para } \theta}$
  - $\theta = \infty$ $f(y, \theta)dy$

- Bayesian

  - $L(\theta) = f(y \theta)p(\theta)$

  - $L(\theta) = C$

- Here $\{f(y, \theta) \mid p(\theta) = a(\theta)\}$

- NB $\{a(\theta) = c\}$

- Bayes is confidence

- Welch & Peers 1963 &

- Examines

- For Statistic, Jeffreys prior: root into prior gives conf to 2nd order

- Says: Scalar Expert model is 1 Coch to 2nd order

- "Math Stat Important!"
Bayes (1763) | Jeffreys (geophysical, prob/math)

\[ \tilde{\theta}(\theta) = \left[ \ln(l(\theta)) \right]^{1/2} \cdot d\theta \]

Jeffreys prior

\[ \pi(\theta; y) = \left[ L(\theta) \left[ \ln(l(\theta)) \right]^{1/2} \cdot d\theta \right] \]

Info/Accuracy

Jeffreys 1964: "Problems" Modified Jeffreys

\[ \frac{\partial N}{\partial \beta} \frac{\partial \beta}{\partial \theta} \]

Nat Haar

What priors? Jeffreys \( p > 1 \)

Jeffreys mod \( \sqrt{\frac{N}{2\pi \sigma^2}} \)

Transform model like \( N \times \beta \sigma^2 \)

Other proposals: -- Bayes literature

Jeffreys \rightarrow \text{Plate Tectonics} \text{ Nonsense}

Fisher \rightarrow \text{Collect data on magnetism; } \vec{f} \in \mathbb{R}^3 \text{; analysis: decision } \rightarrow \text{Plt}

\[ i(\theta) = E_{\tilde{\theta}} \left[ -\ln(l(\tilde{\theta}); y) \right] \]
Normal models; Exponential models (Generalized...just linearly); ... but, but, ...

Better models: no both error & structure. Taylor is available to simplify


Seemingly unrelated regression (SUR) 221.pdf Journ Econometrics Fraser Rekkas Wong

A comment: Use Likratio 2(L-^-hat) and Bartlett correction ?

Get third order! Likratio But lose direction of departure

1111 X

Books

Here: Properties of Expre models ... 2nd order (more flexible) 264

Real life issues: Are you going to be B and throw critical info away ??

Why two "theories"? You need to know! Welch Peers Vector case

EM $f(x; \theta) = \phi(x - \phi) \exp\left\{-\theta_1 \phi^3 n^{1/2} + \theta_2 \phi^3 / 6 n^{1/2}\right\} \left(1 - \theta_3 \phi^2 / 2 n^{1/2}\right)$, as \( \phi \rightarrow \infty \)
Normal models; Exponential models (Generalized... just linearly); ... but, but

Better models: re both error & structure


Seemingly uncorrelated regression (SUR) 221.pdf Jour Economometrics Fraser Rekcas Wong

A comment: Use Likratio 2(\hat{\theta} - \theta) and Bartlett correction!

Get third order!
But lost direction of departure

\[ f(\theta; \varphi) = 1/(\sqrt{2\pi\sigma^2}) \exp\left\{ -\frac{(\theta - \varphi)^2}{2\sigma^2} \right\} \]  
\[ (1 - \alpha \cdot 3\delta/2n^2) \]

Here: properties of Expte models ... 2nd order (more flexible!)

Real life issues: Are you going to be B and throw critical info away? ??

"Our" model for calculations -- 1
But 40 vector!
Laplace Gauss "Liked" flat priors: Easy answers; seemed sensible.

Jeffreys

\[ i(\theta) = \int -\log(\theta; y) f(y; \theta) \, dy = E - \log \]

Jeff \to Bayes \quad \pi(\theta) = i^2(\theta) \quad \text{Scalar } \theta

\[ \text{W-P \ \inference} \quad \int_{-\infty}^{\infty} \pi(\theta) L(\theta) \, d\theta = \int y f(y; \theta) \, dy \quad p(\theta) = \text{"Stoch. Inc"} \]

Use scalar Jeff

95% Int \to 95% CI

\beta \text{ Int} \to \beta \text{ CI}

\text{N.B.} \quad \text{Most of all CI.}

if \ f(y; \theta) = f(y - \theta) \quad \text{Location}

otherwise: Usually different but \text{Exact model}

\text{W-P}\quad \Rightarrow \text{Second order}

\Rightarrow \text{Prove here: easily}

\Rightarrow \text{Easy W-P proof}
Proof of WP.

Prove: Jeffreys for Scalar par exp De model gives Conf Ints

Use: Confidence bounds; use quantile => Also for intervals

Scalar para M

Check Jeff

Jeff:

\[ \beta(\theta) = \beta = \frac{\int_0^\theta (1 + \alpha \theta / 2 \sqrt{n}) d\phi}{\alpha \beta^{2/4} / \sqrt{n}} \]

\[ \theta = \Phi(\beta) \]

\[ \hat{\theta} = \Phi^{-1}(\beta) \]

\[ \Phi(\beta) = \beta \]

\[ \hat{\phi} = \frac{\sum (1 + \alpha \theta / 2 \sqrt{n})}{\alpha \beta^{2/4} / \sqrt{n}} \]

\[ \log L_k (\hat{q}, \hat{p}) \]

\[ \hat{q} = \frac{\hat{p}}{1 - \alpha \hat{p} / 2 \sqrt{n}} \]

1st

\[ \hat{p} = \hat{p} \]

E: Can perf.

Solve Quadv. in \( q \) & perfh.

Score -> MLE

Constant info

repara....
\begin{align*}
\beta &= \int_0^\infty \left[ 1 + \alpha \varphi / 2 \pi n^{1/2} \right] d \varphi = \varphi + \alpha \varphi^2 / 4 n^{1/2} \\
\varphi &= \beta - \alpha \beta^2 / 4 n^{1/2} \\
\beta &= \hat{\beta} + \alpha \hat{\beta}^2 / 4 n^{1/2} \quad \text{Why? Inv. under replica!}
\end{align*}

\begin{align*}
\hat{\beta} &= \beta - \alpha \beta^2 / 4 n^{1/2} + \alpha \hat{\beta}^2 / 4 n^{1/2} = \beta - \alpha \beta^2 / 4 n^{1/2} \\
\beta &= \frac{\hat{\beta} + \alpha \hat{\beta}^2 / 4 n^{1/2}}{1 - \alpha \beta / 2 n^{1/2}} \\
F(n; \varphi) &= \frac{1}{\sqrt{2\pi}} \exp \left\{ - \frac{(n-\bar{\varphi})^2}{2} - \alpha \varphi^3 / 6 n^{1/2} + \alpha \hat{\varphi}^3 / 6 n^{1/2} \right\} (1 - \alpha \beta / 2 n^{1/2})
\end{align*}

\begin{align*}
\varphi &= \beta - \alpha \beta^2 / 4 n^{1/2} \quad \text{Substitute} \\
\beta &= \frac{\hat{\beta} + \alpha \hat{\beta}^2 / 4 n^{1/2}}{1 - \alpha \beta / 2 n^{1/2}} \\
&= \frac{1}{\sqrt{2\pi}} \exp \left\{ - \frac{\beta^2}{2} - \alpha \beta^2 / 12 n^{1/2} \right\} - \beta^2 / 6 n^{1/2} + \alpha \beta^3 / 6 n^{1/2} \\
&= \frac{1}{\sqrt{2\pi}} \exp \left\{ - \frac{\beta^2}{2} - \alpha \beta^2 / 12 n^{1/2} \right\} - \beta^2 / 6 n^{1/2} + \alpha \beta^3 / 6 n^{1/2} \\
&= \frac{1}{\sqrt{2\pi}} \exp \left\{ - \frac{\beta^2}{2} - \alpha \beta^2 / 12 n^{1/2} \right\} - \beta^2 / 6 n^{1/2} + \alpha \beta^3 / 6 n^{1/2}
\end{align*}

\begin{align*}
\int_0^\infty \exp \left\{ - \frac{\beta^2}{2} - \alpha \beta^2 / 12 n^{1/2} \right\} d \beta = \frac{1}{\alpha^{1/2} \beta^{1/2} \sqrt{2\pi}}
\end{align*}

\begin{align*}
\text{Gagn model}
\end{align*}
Taylor expansion:

\[ f(y) = f(y_0) + f'(y_0)(y-y_0) + \frac{f''(y_0)}{2!}(y-y_0)^2 + \ldots \]

\[ y \approx y_0 \quad f(y) = f_0 + f_1(y-y_0) + \frac{f_2}{2!}(y-y_0)^2 + \frac{f_3}{3!}(y-y_0)^3 + \ldots \]

Taylor (Ito)

Tensored notation: sum over indices

\[ f = f_0 + f_1 y_0 + f_2 y_0^2 + \frac{f_3}{2!} y_0 y_1^2 + \ldots \]

\[ x, p = 1, 2, \ldots, p \]

\[ f_{\alpha_1 \alpha_2} \rightarrow \sum_{\alpha_1} f_{\alpha_1 \alpha_2} \quad y_1 y_2 y_3 \sim \mathcal{N} \]

\[ y_1 y_2 y_3 \quad \text{once} \]

\[ y_1 = y_1 y_2 y_3 \quad \text{three times} \]

Taylor's Cosmodel

O(n,h) O(n') terms

CLT → Earle's

Seeing what function of interest looks like locally

Delta method

Current directions!
Where are we?

1. Likelihood (log version)
   \[ \ell(\hat{\theta}; y) = \ln \hat{\ell} - \ln \ell = \frac{n}{2} \]
   \[ n = \frac{\hat{\ell} - \ell}{\sqrt{2}} \]
   \[ \theta = \text{Para. Inv.?} \]

2. Exponential model: Approx for general model! Power!
   \[ f(y; \theta) = c(\theta) e^{\theta y} \]
   \[ h(y) \rightarrow \text{Special} \Rightarrow \text{Gen(Lin)} \text{models} \]
   \[ f(\hat{\theta}; y) = c(\hat{\theta}) e^{\hat{\theta} y} H(\hat{\theta}) \approx \hat{\ell} - \ell \approx \frac{n}{2} \]
   \[ S^2 = \frac{c(\hat{\theta})}{(2\pi)^{p/2}} e^{\hat{\theta} - \ell} \]
   \[ O(n^{-1/2}) \]

3. General A.S. model: \( f(y; \theta) \)
   \[ V = \frac{\partial^2}{\partial \theta^2} \text{fixed } p \text{-vector} \]
   \[ \phi(\theta) = \frac{\partial}{\partial \theta} | y \]
   \[ \text{Get: Exp. Model} \]
Why we study Exptl models? See above! Now: Location vs. Exptl models!

1. Location: $f(y - \theta)$
   - $p(\theta) = \int_{-\infty}^{\infty} f(y - \theta) \, dy$
   - $A(\theta) = \int_{-\infty}^{\infty} f(y - \theta) \, f(y - \omega) \, dy$
   - Vector $\{y_i, \theta\}$

2. Exptl: $c(\theta) \exp\{\phi(\theta) \delta(y)\} f(y)$
   - Marginal to $f(y)$
   - Canonical $\phi(\theta)$ & SP.

Wald–Peers 1963

Scalar $\alpha, \phi$

Centered $\alpha$ & $\phi$

Use $n(\theta) = \frac{\theta^2}{2}$

Get confidence to 2nd order

Proof via 2nd order Taylor

Last day: $E(y^2 \mid \phi) = \phi^2 + 3\phi$ in Normal case

SP but 2nd

Story

$N $0 1

$S$-3s $\rightarrow \phi$

Know: When

264-exl-Student.pdf

$f(\beta - \beta)$

2nd

$f(\hat{\beta} - \beta) = \frac{1}{\sqrt{2\pi} \sigma^2} \exp\left\{-\frac{\hat{\beta}^2 - \beta^2}{2\sigma^2}\right\} \, d\hat{\beta}$

264+exl-Student.pdf 265.pdf (to come)
\( f(y; \theta) = c(\theta) \exp \{ \phi(\theta) \cdot s(y) \} h(y)^{-\frac{1}{2}} \)

\( g(s; \phi) = \frac{1}{(2\pi)^{\frac{n}{2}}} \exp \{ -\frac{1}{2} (s - \hat{\phi})^T \Sigma^{-1} (s - \hat{\phi}) \} \)

1. \( \lambda < \lambda_0 \) \( \phi < \phi - \hat{\phi} \) nice
2. Scale: \( \hat{\phi} = I \) \( \phi \leftarrow \frac{1}{1 + \hat{\phi}} \cdot \phi \)

\( \hat{\phi} = -\frac{1}{2} \left\{ (s_1 - \phi_1)^2 + \ldots + (s_p - \phi_p)^2 \right\} \)

\( \exp \{ -\hat{\phi} \} = \phi(s - \phi) \)

\( \text{In level here!} \)

General \( \lambda \Theta \in \mathbb{R} \) 

\( \text{Thank you!!} \)

\( \text{Next here!} \)
\[ g(x, \phi) \text{ after Stagation} \]
\[
= \phi(x-x) \exp \left\{ -a_{11} \phi_1 \phi_1 / 2n^{1/2} + a_{111} \phi_1 / 6n^{1/2} \right. \\
- a_{12} \phi_1 \phi_2 / n^{1/2} + a_{112} \phi_1 \phi_2 / 2n^{1/2} \\
- a_{13} \phi_1 \phi_3 / n^{1/2} + a_{123} \phi_1 \phi_3 / 2n^{1/2} \\
+ a_{21} \phi_2 \phi_1 / n^{1/2} + a_{22} \phi_2 \phi_2 / 2n^{1/2} \\
+ a_{23} \phi_2 \phi_3 / n^{1/2} + a_{213} \phi_2 \phi_3 / 2n^{1/2} \\
+ a_{31} \phi_3 \phi_1 / n^{1/2} + a_{32} \phi_3 \phi_2 / 2n^{1/2} \\
+ a_{33} \phi_3 \phi_3 / n^{1/2} + a_{312} \phi_3 \phi_2 / 2n^{1/2} \\
\left. \right\} \text{d}x
\]

\[ N \phi \]
\[ E(\lambda) = \phi \]
\[ E(\lambda^2) = \phi^2 + 1 \]
\[ E(\lambda^3) = \phi^3 + 3\phi \]
\[ E(\lambda^4) = \phi^4 + 4\phi^2 + 2 \]

Expand \( \lambda - \bar{\lambda} \) to next order; have \( \phi(x-x) \)
\[
\begin{bmatrix}
\phi_1^2 \\
\phi_1 \phi_2 \\
\phi_1 \phi_3 \\
\phi_2 \phi_3
\end{bmatrix} \begin{bmatrix}
\Delta_1^2 \\
\Delta_1 \Delta_2 \\
\Delta_1 \Delta_3 \\
\Delta_2 \Delta_3
\end{bmatrix}
\]

Have the mult ExpTo model in \( \text{Tray O}(n^{-1}) \) fom

Reading (for details) \( \rightarrow 264 \text{. pdf} \)
Mini project \( f(\beta-\beta) \)
1) 264. pdf: Details: Corrections; Understanding
   Workshop Seminar
   Participation

2) 265... coming! Or...?
   Tools: $E(\theta); \ell(\theta;y);$ Distns; $SP; p^*;$ Exptl models; log-model expansions
   New tools for new problems
   ODE tools for new problems
   New tools for new problems $SP, p^*$ etc as done

3) Questions re "Inference"
   $CLT \Rightarrow \theta \Rightarrow \text{Asy } N(1) \Rightarrow \text{Appx } N \Rightarrow \text{LLN Asy } \hat{\theta} \ldots$
   $n \Rightarrow \text{very a lot}$
   $0(\sqrt{n}) \Rightarrow \alpha(n^{-\frac{3}{2}})$ prec.

265: Use rec. L Anal $\Rightarrow$ prior

2) Rec

$\mathcal{S}_n \rightarrow \chi^2 \left( \phi - \phi_0 \right) \frac{1}{2} \rightarrow \phi$

Triovial $\Rightarrow (1/2) \chi^2$ or the

Pf family

$\hat{\phi}$ for $q_0$ Way back $\hat{\phi} = \frac{1}{2}n \Rightarrow \phi_0$ from

$\sim N(\ldots)$

1st infer

$(\phi - \phi_0) \frac{1}{2} \text{ in } 2.5$ $\text{slch}$

How does (3) $\Rightarrow$ (4)

$\text{Exp} \Rightarrow \text{Exp}(\phi); \phi^* \text{ comes from stat. cali; Expd}$

Recall: Exp$(\phi)$ Exp$(y)$ Elic; Failrate

$\phi(\phi) = F(y; \phi)$ Table

$\phi(\phi) = \int \frac{1}{2} \theta_0 \theta_0 \text{ PDF}$

$\alpha(\theta) = \int_0^1 \frac{1}{2} \theta_0 \theta_0 \text{ PDF}$

$\text{Recall: Exp}(\theta) \rightarrow \text{Exp}(y) \Rightarrow \text{Elct; Failrate}$

$\phi(\phi) = \int_0^1 \frac{1}{2} \theta_0 \theta_0 \text{ PDF}$
\[ f(y \mid \theta) \]

\[ p(\theta) = \int_{-\infty}^{\infty} f(y \mid \theta) \text{d}y \Rightarrow n(\theta) = \int_{-1}^{1} f(\theta \mid \phi) \text{d}\phi \]

Why equal? NB: Limit/heat

\[ \mathcal{N}(0,1) \]

\[ p(\theta) \text{us } a(\theta) \text{ when } f(y \mid \theta) \text{ ex Exp} \]

W-P definitive (asy; 2nd; proof hints)

\[ \text{Adjusted Jeffreys } \quad \text{dep}(\phi) \]

\[ \frac{1}{2} \left( \phi - \phi_0 \right) = \mu u \]

\[ \phi_0 \text{ does not depend on data} \]

\[ \phi = \text{Can/Exp param} \]

\[ \frac{2\gamma}{\phi} \]

\[ \mu \times \times \]

\[ \phi(\phi) = -K(\phi) = \mathcal{G}(\phi) \]

\[ \text{NB: Any model } \rightarrow \text{EM } \& \text{ 3rd mde} \]

\[ O(n^{1/2}) \]

\[ O(n^{-1}) \]

\[ \text{Error acc. } O(n^{1/2}) \]

\[ \text{Prob theory } \]

\[ \Phi(\ldots) = p + O(n^{-1/2}) \]

\[ \text{Rate to "limit"} \]

\[ \frac{X/3}{n \times n} \text{ more } \]

\[ \frac{n^{1/2}}{n !/2} \]

\[ n^{1/2} \]

\[ n^{1/2} \]

\[ n^{1/2} \]

\[ n^{1/2} \text{ HIGH for } n = 2 \]

\[ n^{1/2} \text{ HIGH for } n = 2 \]

\[ \text{MCMC: } 2 \text{ fig} \text{ mean } 3 \times 10^6 \]
Scalar Exp model 2nd $g(0; \phi) \exp\left[-\alpha \phi / h^2 + a \phi^2 / h^2 \right] (1 - \alpha \phi / h) \cdot \text{as} = g(0; \phi)$

$\phi / \phi$ \hspace{1cm} $\phi / \phi$

GLM: Other quantities $a$ asymptotic, layout calc.

$\mathcal{L}^* = N \cdot \mathcal{L} \cdot \log \sqrt{b / q}$

Uses SLR $r$

$mee q$ but cannot

$r^* \in \mathcal{S}P$ -- Not in Daniels


Not just $\mathcal{S}P$

Version using $r^*$ plus isolation of $\phi^2$

Likelihood $G(\phi)$

Link $m$: inverse

$G(\phi)$ = Lik anal $\phi$ $N(\phi, y)$

No 3rd order $r$ near $\phi$

Depend on $\alpha$ or $\phi$

$N / 2$ NB

Staebler

Lik ratio (SLR)
Exp model 2nd Order

\[ g(z; \theta) = \phi(z; \theta) \exp\left\{ -\left( \frac{\theta_1}{2} z^2 + \theta_2 z + \theta_3 \right) \right\} \]

Vector version: \( \theta_1, \theta_2, \theta_3, \ldots \)

Scalar case: Project \( n, \theta, \sigma^2, G \) & more & potential publication [correct!]

Alternatives? 4412 \( \Rightarrow \) Research aims: publishable: A contribution to Statistics

Do here in C/B? First, Deriv

The research!

Score \( \hat{\theta} = 0 \)

\( \hat{\phi}(z) = \frac{a - \theta_0}{2n^{1/2}} \)

Want \( n \cdot \frac{c}{2} \cdot \text{need} \)

\( \hat{e} = -\frac{a^2}{2} + \frac{\theta_0}{2} a^2 + \frac{\theta_1}{6} n^{1/2} + \frac{\theta_2}{2} n^{1/2} + \frac{\theta_3}{2} n^{1/2} \)

\( = \frac{\theta_0}{2} a^2 + \frac{\theta_1}{6} n^{1/2} + \frac{\theta_2}{2} n^{1/2} + \frac{\theta_3}{2} n^{1/2} \)

\( \hat{e} - \frac{n^{1/2}}{2} = (\frac{\theta_0}{2} a^2 + \frac{\theta_1}{6} n^{1/2} + \frac{\theta_2}{2} n^{1/2} + \frac{\theta_3}{2} n^{1/2}) \)

\( = (\frac{\theta_0}{2} a^2 + \frac{\theta_1}{6} n^{1/2} + \frac{\theta_2}{2} n^{1/2} + \frac{\theta_3}{2} n^{1/2}) \)

\( \hat{c} = \frac{a}{2} n^{-1/2} - \frac{\theta_0}{2} a^2 + \frac{\theta_1}{6} n^{1/2} + \frac{\theta_2}{2} n^{1/2} + \frac{\theta_3}{2} n^{1/2} \)

\( \left( x^2 + ax + a^2 \right) = \left( x + \frac{a}{2} \right)^2 \)

\( \sqrt{2} \text{ Root} \)

\( \text{Factors} \)

\( \text{Recall} \)

\( \text{Sign} \) of \( 1 - \phi \)
\[ p = -\phi^2 / 2 - \rho \phi^3 / 6n^{1/2} + \phi \delta \]
\[ \hat{\phi} = s - \alpha s^{1/2} / 2n^{1/2} \]
\[ \hat{\rho} = \phi = s - \alpha s^{1/2} / 2n^{1/2} + \phi \delta \]
\[ \hat{\phi} = s - \alpha s^{1/2} / 2n^{1/2} \]
\[ \hat{e} = \text{SLR} \]

Recall: Expt model \((d_{n+1} = 1)\): General form \( g(\hat{\phi}; \delta) \)

\[ q = (\hat{\phi} - \phi) \int_{\phi}^{1/2} \]

\[ = (s - \alpha s^{1/2} / 2n^{1/2} - \phi) \left( 1 + \alpha s / 2n^{1/2} \right) \]

\[ = s - \phi - \alpha \phi s / 2n^{1/2} \]

\[ - \alpha s^2 / 2n^{1/2} + (s - \phi) \alpha s / 2n^{1/2} \]

\[ r = s - \phi - \alpha (\phi^2 + \phi s + \phi^2) / 6n^{1/2} \]

\[ q = s - \phi - \alpha \phi s / 2n^{1/2} \]

\[ \hat{\phi} = s - \alpha s^{1/2} / 2n^{1/2} \]
\[ r = \alpha - \phi - \alpha(G^2 + \phi G + \phi^2)/6n^{1/2} \]
\[ q = s - q - \alpha \phi s/2n^{1/2} \]
\[ \phi = s - \alpha \phi^2/2n^{1/2} \]

Want now a quotient #

\[ r* = \log \frac{n}{q} \]

\[ \frac{r}{q} = \frac{s - q - \alpha(G^2 + \phi G + \phi^2)/6n^{1/2}}{s - q - \alpha \phi s/2n^{1/2}} = 1 - \alpha(G^2 + \phi G + \phi^2)/6n^{1/2} \]

log \frac{n}{q} = -\alpha(d - q)/6n^{1/2}

\[ r* = s - q - \alpha(G^2 + \phi G + \phi^2)/6n^{1/2} + \alpha/6n^{1/2} \]

Next we calculate

\[ G(0; \mu, \sigma) = \int G(n; \mu, \sigma) \, d\sigma \]

Next we calculate

\[ \Phi(n) = \Phi(\mu) + \sigma(n-\sigma) \int \Phi(\mu) - \phi(x) \, dx \]

\[ g(x, y) = g(x) + g(y) \]

value

2nd

Next we calculate

\[ G(0; \mu, \sigma) = \int G(n; \mu, \sigma) \, d\sigma \]

Get equal

\[ 207, \text{ pdf} \]

\[ J - F \text{ on web} \]

\[ LR \text{ Bartlett} \]

Scale

3nd
Model skewness and its effect on inference

(i) Generalized linear models: Exponential model &
Some parameterization of $f(y_i; \theta)$ is taken to have linear form $X_i \beta$

(ii) Tools

1. Exp model
   2nd order
   $q(s, q) = \phi(s - q) \exp\left[-\alpha \frac{q^2}{2n^{1/2}} - \frac{d}{2n^{1/2}}\right] \left(1 - \frac{q^2}{2n^{1/2}}\right)$. Do
   Vector version: $\alpha_{111}$, $\alpha_{112}$, $\alpha_{123}$
   This is full SP - 2nd order - scalar variable, scalar parameter

2. Background: Laplace, SP, $p^*$, $r^*$, $p(r) = \Phi(r^*)$ Full control (3rd) inference procedure

$r = s - \varphi - \alpha (\varphi^2 + \varphi + \frac{s^2}{2}) / 6n^{1/2}$
$q = s - \varphi - \alpha \varphi s / 2n^{1/2}$
$\hat{\varphi} = s - \alpha s / 2n^{1/2}$

Parameterization in variance:
$q e^{c q y} \leftarrow e^{-q y / \theta} \leftarrow$
MLE departure re can pan $\Rightarrow$ generalizes widely (compare TEM)

3. Bartlett
   Reduced for $s$ after centering
   $f(s, q) = \exp\{q^2 s + l(q)\} h(s)$, $d_s \overset{\text{EM}}{\sim}$ SP $[\text{flexible}]$
\[ \theta = (\psi, \lambda) \]

\[ \varphi \sim \cdots \varphi \]

Describe

1st para no R = \text{SLR} = \pm \left[ \frac{1}{2} (\hat{e}^2 - \bar{e}^2) \right]^{1/2} < \]

\[ \text{Scalar } \gamma \]

\[ \hat{e} = l(\theta) \]

\[ \bar{e} = l(\bar{\theta}) \]

\[ \delta = \arg \max l(\theta) \]

\[ \gamma(\theta) = \gamma_0 \]

\[ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \]

Lin par cp EM

Important Simplicity

\[ \gamma_0 \approx \gamma_0 \]

Possible \( \varphi \) value when \( \gamma = \gamma_0 \)

\[ \hat{\varphi} = \varphi(\hat{\theta}) \]

\[ \varphi \left( \varphi \right) = \varphi_0 \]

Vec

Need a scalar version \( \gamma \)

Orthogonal parameters

- Obs info Current
- Exp info Origin

Can var

\[ \cdots \]
\[ \Theta = (\psi, \lambda) \]

1st para in no. \( \lambda = \frac{1}{2} \left( \hat{L} - \tilde{L} \right) \frac{1}{2} \]

\[ n^* = n - \lambda' \log \% \]

\[ \text{Dev} \lambda < \lambda_0 \Rightarrow \text{plug in sign}(\hat{y} - y_0) \]

Scalar \( \gamma \)

\[ \hat{\gamma} = \gamma(\hat{\theta}) \cap \gamma = \gamma(\theta) \]

\[ \delta = \arg \max \gamma(\theta) \]

\[ \gamma(\theta) = \gamma_0 \]

Orthogonal parameters
- Obs info current in var
- Exp info orig var

Possible \( \phi \) value when \( \gamma = \gamma_0 \)

\[ \phi = \phi_0 \]

Can var

Can var
Parameter $\mu(\theta)$ of interest

got an estimate $\hat{\mu}$ for $\mu$ in lieu of sq.

Hyp: $\mu = \mu_0$

Assess $\mu(\theta)$: cf $\mu_0$

Bootstrap $\hat{\mu}$: Non-Param

You resample your observed errors and thus get new (BS) sample

Calc. Departure measure $\times$ quen

Do $nS$ times $3 \times 10^6 \Rightarrow 10,000$

Obs $\mu$ of BS rep's. BS value

Need LS, MLE, other quen

Convert to Stat term $\# p$ value?

Not $p$ value $U(\theta, \hat{\mu})$?

Accuracy < Right measure?

Conclude a manic.

Departure $\hat{\mu} - \mu_0$:

1st $\mu$

Stat mean of Dep

$G = \frac{[x_\mu - \mu_0]^2}{n}$

Stat Mean

1st $\mu$

$\bar{x} = \frac{1}{n}$

Stat Mean

1st $\mu$

$G = \frac{[x_\mu - \mu_0]^2}{n}$

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$G = \frac{[x_\mu - \mu_0]^2}{n}$

Stat Mean

1st $\mu$

$\bar{x} = \frac{1}{n}$

Stat Mean

1st $\mu$
Parameter $4(y) = y_0$
Estimate $\hat{y}$ by $o(b, b)$

Prognostic simplistic, elementary
$y \sim \theta_0$ Past performance

Est $\theta$ \uparrow CERN LHC

$\hat{y} - y_0$

$[\hat{y} - y_0]/\theta_0$ ?

Stat terms $p$ value \downarrow

Tangent BS etc.

$f(s; \phi) = \phi(s - y) \exp\left\{-\frac{3}{6} s^{3} + \frac{3}{6} s^{3/2}\right\} (1 - \frac{3}{6} \frac{1}{s^{1/2}}) \cdot ds$

EM\textsuperscript{\textdagger}

Simple $p = 1$

Skewness is ignored in SLR or in LR

% Importance

[Get $U(0,1)$ Wrong measure

Read: Questions: Answer
G. LR models
S. Kennedy
Research Topic and The tools of Inference: March 8, 2012

1. Null/alternative hypothesis
2. p-value
3. Reporting results

Skewness

\[ f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{x-\mu}{\sigma} \right)^2} \]

1. **Likelihood Ratio Stat**
2. **Empirical**
3. **Log likelihood**

\[ \Lambda = L(\theta) - L(\hat{\theta}) \]

- Replace and integrating out "nuance"
- Saddlepoint (EM) and it (gen as models)
- \( \exp \left[ -\frac{\hat{\theta}^2}{2\hat{\sigma}^2} + \frac{x^2}{2\hat{\sigma}^2} \right] \)
- \( 1 - \alpha \) in LR
- Summarize
- 2nd moment
- What is it?
- BS prior
- Vector
- 2D
- Asymptotic
- Accurate
- EBM
- Q8

Data: Hedges M, Donohue P

- Test: Not Conf! Approximation of Edelman
- Quick
- Just
- Evidence

* burp

CERN - Explorations

- ADT
- "Hedeb"
Why? 8

Delay on GLH

260 Line-End

Delay in Slewing

N(\text{phi})

Real Invariant 2

Tell about

N/y, z

Delay in Slewing

G/M = Gain block = \text{Xp}

Inverse of them

Identity

Inverse of them

Scalar and normal for Xp

G/M

Log e

Least: \log_2 \frac{1}{1-p}

Continued

Identity

Log e

Least: \log_2 \frac{1}{1-p}

Continued

Identity

Log e

Least: \log_2 \frac{1}{1-p}

Continued

Identity

Log e

Least: \log_2 \frac{1}{1-p}

Continued

Identity

Log e

Least: \log_2 \frac{1}{1-p}

Continued

Identity

Log e

Least: \log_2 \frac{1}{1-p}

Continued

Identity

Log e

Least: \log_2 \frac{1}{1-p}

Continued
What is nice is that we know if $\phi \leftarrow \text{True}$ or False, we can use the confidence region. Given the confidence region, what is $P(\theta | y)$? How is $P(y | \theta, x)$? Could there be a connection? We could come up with all this. How does $P(\theta | y)$ change as $y$ changes? How well does $P(\theta | y)$ change as $y$ changes?

$P_{\theta}(y) = \exp \{ p(y | \theta) - \phi_0 \}$
$$p(y; \theta) = F(y; \theta)$$

Thoughts

Accept / Reject

Newman-Pearson (More Later)

$$\text{Pr}u = \{y_1, \ldots, y_n\}$$

Example:

$$f(y_1, y_2; \phi_1, \phi_2) = \exp\left[\phi_1 y_1 + \phi_2 y_2 - K(y_1, y_2)\right] / \gamma(0_1, 0_2).$$

$$= \frac{e^{\phi_1 y_1}}{2\pi} \cdot \frac{e^{\phi_2 y_2}}{2\pi} \cdot \frac{1}{\gamma(0_1, 0_2)^{1/2}} .$$

Integrated from a MSS $\{y_1, y_2\}$

$$S'F(3)$$

A lot of nice properties

Parameters: $\phi = (\beta_1, \ldots, \beta_n)$

Model in the MSS (Traditional)

$$N(\mu, \sigma^2) \sim N(X\beta, \sigma^2)$$

Few more cases

Generalized

Do??
GLM \[ \exp\{ (x_\beta)'s + k(x_\beta) \} R(s) \]

\[
\begin{align*}
y_1 \sim & \chi_1 \beta_1 \\
y_n \sim & \chi_n \beta_n \\
\text{Interest in } \beta_n \text{ and Interest in } x
\end{align*}
\]

Interest in x: Whole story (!) is on Line/Slice L / 3rd

Linear x

Background: Neyman-Pearson fight Fisher
Test of Hyp (composite) is nuisance parameter

Exterior models; Completeness.

EM \[ \exp\{ x_\beta' + z_\beta - k(x, \beta) \} R(\beta, t) \]

Test \[ x = x^0 \]

UMP similar which happen to be free

\[ \text{procedure } \beta \text{ nuisance parameters} \]

260 answer Lin Par
4 412.12 18

Diagram:

\[ \phi = \frac{\phi(\hat{\beta} \epsilon)}{\phi(\epsilon)} \]

General \[ \phi \rightarrow \beta \]

\[ x = \hat{\beta} \epsilon \]

Lehne & Romano
Model: \( f(s_1, s_2; \varphi_1, \varphi_2) = e^{s_1 \varphi_1 + s_2 \varphi_2 - K(s_1, s_2)} \frac{h(s_1, s_2)}{h(s_1, s_2)} \) via SP. \( \varphi_1 = 4 \), \( \varphi_2 = 2 \)

Q Distn of \( s_1 | s_2 \) is just \( s_2 \) section of model, \( f(s_1 | s_2) = e^{s_1 \varphi_1} h(s_1, s_2) \times \text{Norm} \)

No \( \psi_2 \) present! No nuisance

Exptl model: Can int. param \( \psi \)

\( f = e^{\psi s_1 + \psi t + k(s_1)} h(s_1, t) \)

\( t = t^* \) section. No new information.

Next day: Check 260: Be prepared.
More on the "Core": Exponential Models

Mainstream reference: Neyman-Pearson 1933

Lehmann & Romano 2005 § 4.4

UMPU & UMP

Current directions!

Q: Unbiased: similar Talk

Example

** f(s, t; y, λ) = d t  

Easy to analyze

= \exp \left[ \frac{y \lambda + \lambda s}{\text{pdf}} \right] h(0, t) dt

= \exp \left[ \frac{\lambda (y \lambda + \lambda s) / 2}{\text{pdf}} \right] \exp \left[ \frac{\lambda (0, t)}{\text{pdf}} \right]

Interest in \( y, t \) ?

\[ \Delta t = t^0 \] gives UMPU/UMP

But: is there more? L + R

\[ \Delta t = t^0 \] depends only on \( y \) No nuisance! No \( \Lambda \)!

Note: power

Let \( X \) vary: Find how interest in \( \Lambda \) is measured
What is xxx? xxx: Unb; xxx is similar

Ortho parameters

Note: Nuisance parameters are occasionally difficult; convenience is arbitrary.

(a) Helpful: \( I(y, x) = E\{ l(x, y); y, x \} \)

If \( \text{cov} = 0 \) at some \( y \); then mechanics can be easier.

(b) \( \uparrow (y, x) \) with \( \text{cov} = 0 \)

Fundamental

Parameter \( y \) and \( x \); \( y \) is interest

Nuis \( \mathcal{N} \) \( y \) \( y' \) \( y'' \)

Linear here on \( \Phi \) space

\( \Phi \) space: Can par. \( y \) can par.

Linear nuis

Linear nuis

\( \Phi \rightarrow \) desc.

\( \Phi \rightarrow \) desc.

Nuisance parameter: \( \mathcal{N} (\mu, \sigma^2) \)

\( \mathcal{N} (\mu_0, \hat{\sigma}^2) \)

\( \mathcal{N} (\frac{\mu}{\sigma^2}, \frac{1}{\sigma^2}) \)
Two versions; each important. Tell different things.

\[ f(s^0, \lambda) = \exp \left\{ \frac{\lambda^0}{\Delta_0} \right\} h(s^0, \lambda) \]

\[ f(s^0, \lambda) = \exp \left\{ \frac{\lambda^0}{\Delta_0} \right\} h(s^0, \lambda) \]

Normal only if

Norm constant

What

\[ f(y, x) = \frac{e^{-\frac{(y-x)^2}{2}}}{\sqrt{2\pi} \sigma^2} \int_{-\infty}^{\infty} e^{-\frac{(y-x)^2}{2\sigma^2}} dy \]
$$f = \exp \left[ \frac{\lambda(x, y) + \lambda(t) + \lambda(z)}{2} \right] k(x, t) \text{ as } t \to 0$$

$$= \exp \left[ - \frac{\lambda^2}{2} = k = \frac{\lambda}{2} \right] \text{ as } t \to 0$$

GO: $\lambda(t) \text{ i.e. } \lambda(t) \to 0$,

$\lambda(t) \text{ is free}$

---

**So space** $\rightarrow$ **$\varphi$ space**

**Linked by $\varphi + \lambda t$**
\[ f = \exp \left\{ \frac{1}{2} \left( \mu(t, \lambda) + \lambda(t, \lambda) \right) \right\} \rho(\theta, t) \, d\mu \]

\[ \rho(\theta, t) = \exp \left\{ -\frac{\theta^2}{2} - \theta \right\} \, d\mu \]

Go: \( \alpha(t) \) i.e., \( \alpha(t^0) \)

\[ \alpha(t^0) = c \exp \left\{ -\frac{\lambda(t^0)}{2} \right\} \int_{\mathbb{R}^n} \rho(\theta, t) \, d\mu \]

\[ \hat{\lambda} = \text{free of } \lambda \]

\[ \hat{\lambda} = \frac{\lambda(t^0) + 2\lambda(t)}{2} - \lambda(t) \]

\[ \hat{\lambda}(t^0; \lambda, t) = \hat{\lambda}(t^0; \lambda) \]

\[ \hat{\lambda}(t^0; \lambda, t) = \hat{\lambda}(t^0; \lambda) + 0 \]

\[ \hat{\lambda}(t^0; \lambda, t) = \hat{\lambda}(t^0; \lambda) + 0 \]

For constant/free of \( \lambda \) \( \Rightarrow \hat{\lambda} \]

Use \( \lambda = \hat{\lambda}(t) \)

Exponential model

Tied to notationally simplified

\[ \hat{\lambda} = \text{fn of } Y \& t \]

\[ \hat{\lambda} = \text{constant along } L \]

for each \( Y \) value

\[ \int \hat{\lambda}(Y, \eta; t, \theta) = \int \hat{\lambda}(Y, \eta) \]

\[ \int \lambda(Y, \eta; t, \theta) = \int \lambda(Y, \eta) \]
For (2,2) Exp (full) "L_0(y) = \text{Full} ; \theta \rightarrow \hat{\theta}_y"; profile of \( y \) along (widely used/widely abused: Good Bad) 

\[
\ell(y; \hat{\theta}_y; s, t) = \ell(y, \hat{\theta}_y; s, t) = \ell(y, \hat{\theta}; s, t)
\]

\[\frac{\Delta^2}{2} = \ell(y, \hat{\theta}; s, t^0) - \ell(y, \theta; s, t^0) = \ell(y, \hat{\theta}; s, t^0) - \ell(y, \hat{\theta}_0; s, t^0) = LR \text{ for testing } \theta_0 \text{ different from } \theta_0.
\]

\[\ell(y, \theta; s) - \ell(y, \hat{\theta}_0; s) \geq 2 \frac{\Delta^2}{2} \text{ (modified formula in data)} \]

\[f(y; \theta_0; s) = \frac{e^k}{\sqrt{2\pi}} e^{-\frac{(y - \mu)^2}{2\sigma^2}} \]

\[\ell' = \text{Ram} \]

Used: 2 versions (Exact & SP); Selected to get \( s|t^0 \); Use SP version one down

NB Exp full model \( s|y = \hat{\theta} : \text{dim } \hat{\theta} = \text{dim } y - d \): 2nd Interest in \( y \) (linear) Get 

Immediate SP Inverse of profile likelihood for Interest \( y \) (linear) SP version

Interest in \( y \) = SP at nuisance

Example 260
Summary

\[ f = \exp \left\{ \ell(\theta, x) + \lambda x + \lambda^T \right\} h(0, t) \cdot \text{do}\ 
\]

Interest in Linear vs Latin in Curved \( \gamma \)

**Expl:** Weibull general!

Any model; 2nd: Fan EM

\[ f(A_{10}) = \frac{e^{\frac{1}{2}}}{\sqrt{2\pi}} e^{-\frac{1}{2} \| A_{10} - \mu \|^2} I_{[\mu]} I_{\left( \frac{1}{2} \right)} \]

Fill 3rd order

Leh Ram approx

# death (not in literature)

\[ \text{Proof} \rightarrow 260 \]

\[ \text{Voluntary} \]

10

11

12

Same as #

Both Condit & Marginal
4. \( f(A; \theta) = \mathcal{O}(A - \theta) \exp(-x_0^2/6) + x_0^2/6 \) for \( 1 - \alpha \leq x_0 \leq \alpha \).

3. Same for weak linear \( y = \beta_0 \) line through data.

2. On \( \theta \to A \) for \( \theta^0 \to 0 \) to \( \theta^0 \to \infty \) one-one mapping.

1. For a linear function \( y = \beta_0 \) through data \( \beta = \beta_0 + \delta \beta \).

\[ \text{log model of } (3n) \]

\[ E_{11} c = 0 \Rightarrow \text{fixed} \]

\[ E_{21} c = 0 \Rightarrow \text{fixed} \]

\[ \text{log model of } (2n) \]

\[ E_{11} c = 0 \Rightarrow \text{fixed} \]

\[ E_{12} c = 0 \Rightarrow \text{fixed} \]

\[ E_{21} c = 0 \Rightarrow \text{fixed} \]

\[ \text{log model of } (2n) \]

\[ E_{11} c = 0 \Rightarrow \text{fixed} \]

\[ E_{12} c = 0 \Rightarrow \text{fixed} \]

\[ E_{21} c = 0 \Rightarrow \text{fixed} \]
1. **Exponential Model**
   \[ f(\theta; \phi) \]
   Test scalar \( \theta \) against \( \chi^2 \)
   Use control model

   Vector \( \Theta; \) Test a commercial (vector) parameter
   \[ f(\theta_1, \theta_2, \theta_3; \phi_1, \phi_2, \phi_3) \]
   \[ \text{dim } 3 \]

   \[ H_0: \chi_1 = \chi_{10} \]
   \[ \chi_{x_2} = \chi_{20} \]

   *Ex: Cont Tables: Independents of rows & columns*

2. **Generalize:**
   Exp Model
   (Next week)

3. **Generalize: General model**
   Case \( \text{dim } \chi = 1 \)
   Similar correspondence
   Case \( \text{dim } \chi = 0 \)
   Define sample space dist.

---

**Easy**

**Easy theory**

---

**Int**

\[ \text{dim } \chi = 2 \]
\[ \text{dim } \chi = 1 \]
\[ p - d \]

---

*Bayesian\# freq calibration*
\[ N m \sigma^2 \]

\[ \frac{1}{\sigma^2} v \]

Not \( \mu \)

\[ \frac{\mu}{\sigma^2} v \]

Not \( \sigma \)

**Explain**

**Int. pan**

Nuis pos (mult; ortho)

**Locn** \( \Rightarrow \) **Bus**

**Impact**

Linear

Curved

Who cares

Accuracy

**N easy (not a good role model)**

\[ mle \]

\[ \hat{\theta} \]

\[ \hat{\theta} = \hat{\beta} \]

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But for Bayesian theory, \( \mu, \sigma \) are linear.

**Location linears**

Difficult

**Exple linearity**: N \& P 1930s; Efron (definition)

**Phys** apart

Impact

With money

Thrust more

Stat issue

GLM

\[ \hat{\beta} \]

\[ \hat{\theta} \]

\[ \text{curve?} \]

Theory

BS, Lik (profile is bad)

NSW, \( \text{3000; literature} \)

[You should know]
\[ \text{lin pan } \frac{1}{\sigma^2} v \rightarrow \text{N}(0,0) \]

\[ \text{int pan } \frac{1}{\sigma^2} v \rightarrow \text{N}(0,0) \]

Explained

But for Bayesian- Frequentist \( \mu, \sigma \) are linear

Location linearity: Difficult

Example linearity: \( N \sim P \text{1980s; Efron (definitive)} \)

Locn \( \Rightarrow \text{Bush} \)

Impact

N easy (not a good role model)

\[ \text{mle } \hat{\Theta}_2 = \hat{\theta}_4 \]

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