Basics: Case: \( y, \theta \) have dim = 1. Pure case.

a) \( f(y-\theta) \)

\[ p(\theta) = \int_{-\infty}^{\infty} f(y-\theta) \, dy \]
\[ \phi(\theta) = \int_{-\infty}^{\infty} f(y-\theta) \, d\theta \]

\[ = F^0(\theta) \]

\[ = c \int \pi(\theta) L^0(\theta) \, d\theta \]

Equal if \( \pi(\theta) = 1 \)

OK as a

convenient C1
Case: \( y, \theta \) have \( \text{dim} = 1 \)

a) \( f(y; \theta) \quad p(\theta) = \int_{\infty}^{y} f(y; \theta) \, dy \)

\[ s(\theta) = \int_{\infty}^{\infty} f(y^0; \theta) \, d\theta \]
\[ = c \int_{\infty}^{\infty} \pi(\theta) L^0(\theta) \, d\theta \]
\[ \frac{\text{Equal if}}{\pi(\theta) = 1} \]

b) \( f(y; \theta) \quad p(\theta) = \int_{0}^{y} f(y; \theta) \)

but Bayes? \( \rightarrow \) (Sec 26.5-158.pdf)

Bayes enigma

Comments? Partisan!

American Scientist 2014 Dec 0/102 p460-465
A. Gelman The Statistical Crisis in Science
Case: \( y, \theta \) have \( \text{dim} = 1 \)

\( a) \ f(y; \theta) \quad \pi(\theta) = \int_{-\infty}^{y} f(y; \theta) \, dy \)

\( b) \ f(y; \theta) \quad \pi(\theta) = \int_{-\infty}^{\infty} f(y; \theta) \, d\theta \)

\[ \begin{align*}
    L(\theta) &= \int f(y; \theta) \, d\theta \\
    \pi(\theta) &= \frac{c}{\int \pi(\theta) L(\theta) \, d\theta} \\
    \pi(\theta) &= 1
\end{align*} \]

"Bayes?" (i) Random \( \theta \)

(ii) "Make up" \( \pi(\theta) \)

(iii) Subjective/opinion

"Genuine" (Efron 2013)

Laplace ""

\( \text{Science} \)
Case: $y, \theta$ have $\dim = 1$

(a) $f(y; \theta) \quad p(\theta) = \int_{-\infty}^{\infty} f(y; \theta) \, dy$

$$d(\theta) = \int_{-\infty}^{\infty} f(y^0; \theta) \, d\theta = c \int_{\theta} \pi(\theta) L^0(\theta) \, d\theta$$

Equal if $\pi(\theta) = 1$

(b) $f(y; \theta) \quad p(\theta) = \int_{-\infty}^{\infty} f(y; \theta) \, d\theta$

"Bayes? (i) Random $\theta$ "Genuine" Efron 2013
(ii) "Make up $\pi(\theta)$" Laplace"
(iii) Subjective/opinion?

(c) $q(y; \theta) \quad \text{Exponential} \varphi(\theta)$

- $\phi(r) \frac{n}{g} \, dr \quad r = \text{SLR}$
- $\phi(r^*) \, dr^* \quad n^+ = r - \bar{r} \log \frac{n}{q}$

$O(n^{-3/2})$

$g = (\hat{\mu} - \mu)^2_{\text{4q}}$

Bayes?

Jeffreys?

265-108.pdf
\[ f(y; \theta) = g(y; \theta) \alpha(\theta) \]

\[ p(\theta) = \frac{L(y; \theta)}{L(\theta)} \]

\[ \text{Exponential } \phi(\theta) \]

\[ (\text{see 265-268.pdf}) \]

1. **Bayes?**
   2. **Jeffreys?**
   3. **Laplace**

Case: \( y, \theta \) have dim = 1

\[ \int_{0}^{\infty} \mathcal{N}(\theta - \mu, \Sigma) \exp \left( -\frac{1}{2} \frac{(y - \mu)^2}{\lambda} \right) \]
More generally? All comes down to dimension 1!

Examples dim.of: $y \sim \text{Stud}(\tau) \mu$

$y \sim \text{EV} \mu \sigma$

$y \sim X \beta + \epsilon \sim z; \text{Stud}(\tau)$
More generally? All comes down to dimension 1!

<table>
<thead>
<tr>
<th>Examples</th>
<th>dim. of</th>
<th>$y$</th>
<th>$\Theta$</th>
<th>$\psi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y \sim \text{Stud}(r) \mu$</td>
<td>$n$</td>
<td>1</td>
<td>1</td>
<td>for $\mu$</td>
</tr>
<tr>
<td>$y \sim \text{EV} \mu \sigma$</td>
<td>$n$</td>
<td>2</td>
<td>1</td>
<td>for $\mu$</td>
</tr>
<tr>
<td>$y \sim x\beta + \xi \gamma$</td>
<td>$n$</td>
<td>$r+1$</td>
<td>1</td>
<td>sure $\beta$</td>
</tr>
</tbody>
</table>

$z \sim \text{Stud}(r)$
More generally? All comes down to dimension 1.

Examples, dim of:

\[ y \sim \text{Stud}(\tau, \mu) \]
\[ n \quad 1 \quad 1 \quad \text{for } \mu \]

\[ y \sim \text{EV} \mu \sigma^2 \]
\[ n \quad 2 \quad 1 \quad \text{for } \mu \]

\[ y \sim \chi^2 \beta + t \xi \]
\[ n \quad n+1 \quad 1 \quad \text{say re } \beta \]

General:

\[ n \quad p \quad d \quad \text{often } 1 \]

\[ T \quad B \quad 1 \quad b \]

\[ t \quad \text{for } t-1 \]
More generally? All comes down to dimension 1!

Examples  dim of
\[ y \sim \text{Stud}(?) \mu \]
\[ y \sim \text{EV} \mu \sigma^2 \]
\[ y \sim x \beta + \varepsilon \sim \text{Stud}(?) \]
\[ \begin{array}{c|c|c|c}
  y & \theta & \mu & \nu \\
  n & 1 & 1 & \text{for } \mu \\
  n & 2 & 1 & \text{for } \mu \\
  n & n + 1 & 1 & \text{say re } \beta \nu \\
\end{array} \]

General

How?

Power transfer

Conditioning Marginalization

Suff. 

How to choose \( \lambda \)
More generally? All comes down to dimension 1!

<table>
<thead>
<tr>
<th>Examples</th>
<th>dim</th>
<th>y</th>
<th>θ</th>
<th>β</th>
<th>ζ</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y \sim \text{Stud}(\tau) \mu$</td>
<td>n</td>
<td>1</td>
<td>1</td>
<td>for $\mu$</td>
<td></td>
</tr>
<tr>
<td>$y \sim \text{EV} \mu \sigma^2$</td>
<td>n</td>
<td>2</td>
<td>1</td>
<td>for $\mu$</td>
<td></td>
</tr>
<tr>
<td>$y = X \beta + \epsilon \sim \text{Stud}(\tau)$</td>
<td>n</td>
<td>$r+1$</td>
<td>1</td>
<td>say re $\beta_n$</td>
<td></td>
</tr>
</tbody>
</table>

General

| n | p | d | often 1 |

How?

- Conditioning
- Marginalization
- Suffey
Conditioning? An example

\[ y_1, y_2 \sim U(\theta \pm 1) \]

\[ y_1 = \theta + 2z \]
\[ y_2 = \theta + 2z \]

how to bring \( \dim y = 2 \) down to \( \dim \theta = 1 \)?

Is there a sufficient statistic? \( \text{NO} \)

\[ U(0-1, 0+1) \]

Error about Examine: little but in every,

\[ y_1, y_2 \sim U(0, \theta) \]
\[ y_1, y_2 \sim U(\theta \pm 1) \]

Sufficient \( N \) is
Conditioning: An example

\[ y_1, y_2 \sim U(\theta \pm 1) \]

\[ y_1 = \theta + z_1 \]
\[ y_2 = \theta + z_2 \]

How to bring \( \text{dim } y = 2 \) down to \( \text{dim } \theta = 1 \)?

\[ \begin{array}{c}
0.1 \\
0.2 \\
0.3 \\
0.4 \\
\end{array} \]

\[ \begin{array}{c}
0.1 \\
0.2 \\
0.3 \\
0.4 \\
\end{array} \]

\[ g(z) = \frac{1}{4} \text{ on } [0, 1] \]
Conditioning: An example

\( y_1, y_2 \sim U(\theta \pm 1) \)

\[ y_1 = \theta + z_1 \]
\[ y_2 = \theta + z_2 \]

How to bring \( \dim y = 2 \) down to \( \dim \theta = 1 \)?
Conditioning: An example

\[ y_1, y_2 \sim U(\theta \pm 1) \]
\[ y_1 = \theta + z_1 \]
\[ y_2 = \theta + z_2 \]

How to bring \( \text{dim } y = 2 \) down to \( \text{dim } \theta = 1 \)?

\[ L^0(\cdot) \]

\[ L^0(\theta) = \begin{cases} \frac{3}{4} & \text{if } \max y_i < \theta + 1 \text{ and } \min y_i > \theta - 1 \\ 0 & \text{otherwise} \end{cases} \]
Conditioning: An example

$y_1, y_2 \sim U(\theta \pm 1)$

$y_1 = \theta + 2.1$

$y_2 = \theta + 2.2$

How to bring dim $y = 2$ down to dim $\theta = 1$?

$L^o(\theta) = \frac{c}{4}$

$max y_\ell < \theta + 1$

$min y_\ell > \theta - 1$

$n \rightarrow P$

Conduction
Conditioning: An example

\[ y_1, y_2 \sim U(\Theta \pm 1) \]

\[ y_1 = \Theta + z_1 \]
\[ y_2 = \Theta + z_2 \]

How to bring \( \dim y = 2 \) down to \( \dim \Theta = 1 \)?

\[ L(\theta) = \frac{3}{4} \left( \begin{array}{c} \max y_i < \Theta + 1 \\ \min y_i > \Theta - 1 \end{array} \right) \]
\[ = 0 \quad \text{O/W} \]

\[ \text{Range of } \Theta = 2 - (\max y_i - \min y_i) = R \]

Depends on data!
Conditioning: An example

$y_1, y_2 \sim U(\theta + 1)$

$y_1 = \theta + z_1$
$y_2 = \theta + z_2$

How to bring dim $y = 2$ down to dim $\theta = 1$?

Another look:
New rotated axes

$g(z) = \frac{1}{4}$ on $\square$
Conditioning: An example

$y_1, y_2 \sim U(\theta \pm 1)$

$y_1 = \theta + z_1$
$y_2 = \theta + z_2$

How to bring dim $y = 2$ down to dim $\theta = 1$?

Another look:

New rotated axes

In $\theta$ direction: $s = \frac{y_1 + y_2}{\sqrt{2}}$

Other directions: $a = \frac{-y_1 + y_2}{\sqrt{2}}$
Conditioning: An example

\[ y_1, y_2 \sim U(\Theta \pm 1) \]

\[ y_1 = \Theta + z_1 \]
\[ y_2 = \Theta + z_2 \]

How to bring \( \text{dim } y = 2 \) down to \( \text{dim } \Theta = 1 \)?

Another look:

New rotated axes

In \( \Theta \) direction:
\[ a = \frac{y_1 + y_2}{\sqrt{2}} \]

Other direction:
\[ a = -\frac{-y_1 + y_2}{\sqrt{2}} \]

What does the model look like \( a \)?
Conditioning: An example

$y_1, y_2 \sim U(\theta \pm 1)$

$y_1 = \theta + z_1$

$y_2 = \theta + z_2$

how to bring dim $y = 2$ down to dim $\theta = 1$?

Another look:

New rotated axes

Marginal for $\alpha$

$g(z) = \frac{1}{4}$ on $\square$

$= 0$ off $\square$

1) New
2) Jac
3) Sori
Another look: New rotated axes

Marginal for $a$

$f(a)$

$-\theta_2$  0  $+\theta_2$

Unif[$\frac{1}{2}(\theta - \frac{\theta}{2})$]

Conrad ola

$g(\theta | a)$

Story
Conditioning: An example

\[ y_1, y_2 \sim U(\theta \pm 1) \]

\[ y_1 = \theta + z_1 \]
\[ y_2 = \theta + z_2 \]

How to bring dim \( y = 2 \) down to dim \( \theta = 1 \)?

Another look:
New rotated axes

Marginal for \( \alpha \)

Condend \( a | \alpha \)

NB: \( a \) has fixed distn and indicates accuracy of inference \( \theta \)
Conditioning

Consider $a(y)$ double $\Theta$-free $q(a)da$

$s | a$ has dim of $\Theta$ $f(s; a; \Theta)ds$

$dm_a = dm \Theta$

Call $a(y)$ ancillary

Bad name: Arbitrary? Apparent arbitrariness
Conditioning

Consider a(y) \text{ with } \theta \text{-free } g(a) da
\Rightarrow a \text{ has dim. of } \theta \ f(s, a; \theta) ds

Call a(y) an auxiliary

Principle (Conditionality / auxiliary)

Use f(s, a; \theta) for inference re \theta

Amelior...

\text{Let } \text{Push } \text{ No!}
\text{Cond. Pple } \text{Yes?}
\text{How to calculate?}
Conditioning
Consider \( a(y) \) distributed \( \Theta \)-free \( g(a) \) \( da \)
\( \mathbf{S} | \mathbf{a} \) has dim. of \( \Theta \) \( f(s|\mathbf{a};\Theta) \) \( ds \)
Call \( a(y) \) **ancillary**

**Principle (Conditionality / ancillary)**
Use \( f(s|\mathbf{a};\Theta) \) for inference re \( \Theta \)

Assume: variable \( a(y) \) inherits continuity in model \( \mathbf{w} \)
Conditioning

Consider \( a(y) \) be \( \Theta \)-free \( g(a) \) \( da \)

* \( a \) has dirn. of \( \Theta \) \( f(a; \theta) \) \( ds \)

Call \( a(y) \) **ancillary**

**Principle (Conditionality / ancillary)**

Use \( f(a; \theta) \) for inference re \( \Theta \)

Assume: Variable \( a(y) \) inherits continuity in model.

Example: \( y_1, \ldots, y_n \sim U(\theta \pm 1) \)

[Change \( \theta \) and examine effect at/near \( \theta \)]  

Continuity Arbukasov
Preamble (to come)

Contour of conditional statistic

\[ y^o = y(\theta; z) \] Quantile from

\[ Fn(y_n) : y_n = y_n(\theta, z_n) \]

\[ \frac{dy}{dy^o} \]

Power

\[ a(y) = a(y^o) \] is local near data point

Invert to get value

What is condition on \( a \)?

Full statistic \( a \)?

No

\[ a(y) \neq a(y^o) \] for small
Q: \( y_1, y_2 \sim U(\theta \pm 1) \) Reference

Can (in general) one quantify "accuracy"?

Yes \( V = \frac{dy}{d\theta} \bigg|_{\theta^0} \)

\[ y = f \begin{pmatrix} \theta \end{pmatrix} \]

\[ \text{plane vector} \]

\[ \mathbb{R}^3 \]

\[ V = (V_1, ..., V_p) \]

\[ L(y) + y^0 \]

\[ \text{TgT plain to } a(y) = a^0 \]

\[ r = \mathcal{R} - \mathcal{F} \]

\[ q_f = \text{a little more} \]

\[ \parallel \]

\[ \parallel \]