Recall: \[ f(t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} \]

Laplace transform:

\[ \mathcal{L}\{f(t)\} = \frac{1}{\sqrt{2\pi}} e^{-\frac{s^2}{2}} \]

Properties:

(i) \[ \mathcal{L}\{e^{-at}f(t)\} = \frac{1}{s-a} \mathcal{L}\{f(t)\} \]

(ii) \[ \mathcal{L}\{tf(t)\} = -\frac{d}{ds} \mathcal{L}\{f(t)\} \]

For a Normal distribution \( X \sim N(\mu, \sigma^2) \), the asymptotic expansion is:

\[ \phi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} + \frac{\mu}{\sqrt{2\pi\sigma^2}} z + \frac{z^2}{2\sigma^2} + \frac{z^4}{24\sigma^4} + \cdots \]
Recall:

1. Laplace integration $f_n(y) = C e^{g_n(y)}$ "Asymptotic $r e n" \text{ Want: Norm} \\
\int f_n(y) \, dy = e^{k/n} \left( 2\pi \right)^{1/2} \int_{y_0}^{y} f_n(y) \\

Uses:

(i) $e^{-\frac{3a_4 + 5a_6}{24n}} \phi(2) e^{a_4 \frac{z}{24n} + a_4 z / 24n}$ is normed

(ii) $\int e^{-\frac{(y - \mu)^2}{2\sigma^2}} = \left( \frac{2\pi}{2\sigma} \right)^{1/2} \int_{y_0}^{y} \phi(2)$

$p$-dim $\int f_n(y) \, dy = e^{k/n} \left( 2\pi \right)^{1/2} \int_{y_0}^{y} f_n(y)$

Hessian at max $d) \log f_n(y)$
Recall

1. Laplace integration: \[ f_n(y) = Ce^{\frac{y}{n}} \] \[ \int f_n(y) \, dy = e^{\frac{y^2}{2n}} \] Full integral

Uses: 1) \[ e^{\frac{3a^2 + 45a}{24n}} \phi(2) \] \[ a_2 \frac{1}{2} / n^2 + a_4 \frac{y}{24n} \] is normed

2) \[ \int e^{-\frac{(y-x)^2}{2n}} \, dy = \sqrt{2\pi} \delta = \sqrt{2\pi} \int y \, dy \]

\[ \text{pdf} \]

P-dim \[ \int f_n(y) \, dy = e^{\frac{y^2}{2n}} \] \[ \frac{1}{\sqrt{2\pi}} \] \[ \frac{y}{\sqrt{y}} \] \[ \frac{f_n(y)}{f_n(y)} \]

Normed version of \( f_n(y) \) Div by \( \int \) \[ \frac{e^{\frac{y^2}{n}}}{\sqrt{2\pi}} \] \[ \frac{f_n(y)}{f_n(y)} \]

\[ \text{Normed pdf} \]

Often: other var \( \alpha \) \[ \theta/n - 1 + \frac{\alpha}{n} \]
Problem 1, 2 of 4

Problem 1: \( \frac{\partial^2 \phi}{\partial t^2} = \nabla^2 \phi \)

Equation of motion:

Use Laplace transform:

Laplace:

Laplace:

Laplace:

\[ E - f = (\phi_1(x) - \phi_2(x)) \]

\[ f(x) \]

Step 1: Assume \( f(x) \geq 0 \)

Step 2: Assume \( f(x) \leq 0 \)

Step 3: Assume \( f(x) = 0 \)

Step 4: Assume \( f(x) \neq 0 \)

\[ \int_{-\infty}^{\infty} f(x) \text{d}x = 0 \]

\[ \int_{-\infty}^{\infty} f(x) \text{d}x = \infty \]

\[ \int_{-\infty}^{\infty} f(x) \text{d}x = 0 \]

\[ \int_{-\infty}^{\infty} f(x) \text{d}x = \text{finite} \]

Exercise 1: \( \text{Exterior} \Rightarrow \text{Scattering point} \)

Exercise 2: \( \text{Exterior} \Rightarrow \text{Scattering point} \)
How to get $SP$ in terms of $r$ and $\phi$. Scalar case

$\frac{\partial f}{\partial u} = -e^{-1/2} \frac{1}{\sqrt{2\pi}} du$

$\text{Scalar SP}$ with change of variable from $u$ to $r$.

$\frac{\partial f}{\partial u} = e^{-\phi(r)/2} \frac{e^{\phi(r)/2}}{\sqrt{2\pi}} dr$

Start with $\frac{1}{\sqrt{2\pi}} e^{-1/2} \frac{1}{\sqrt{2\pi}} du$

$\frac{\partial f}{\partial \phi} = \Phi(\phi; u) \frac{e^{\phi(r)/2}}{\sqrt{2\pi}} dr$

Scalar $SP$ with

$\text{Perturbation is } N(0, 1)$

Diff're data vary $u$ and note $\phi' = \Phi'(u)$

Want $u \rightarrow r$

Diff're $\phi$ variable

Score $\Phi(\phi; u) = 0$
\[
\frac{\alpha}{q} = 1 + a_1 \frac{n}{\sqrt{n}} + a_2 \frac{\log n}{\sqrt{n}}
\]

Coming soon:

1) Assignment 2

2) Date for Test

Variable carrier

\[ C(\Theta) = \{ y_i : f(y_i ; \Theta) > 0 \} \]

\[ y_1, \ldots, y_n \quad \text{max} \quad y_i = \text{MSE} \]

 Highly accurate

Our attention: \( C(\Theta) \) is \( \Theta \)-free

Fixed carrier

Exp Model \( \phi = (\phi_1, \ldots, \phi_p) \)

\( \phi \) exists; unique
Analysis: $\log f(y; \theta)$ Expand by Taylor

$E_M \theta = r + a \frac{\theta^2}{2n} + b \frac{\theta^3}{6n}$

$log \frac{d}{d \theta} = a \frac{\theta}{2n} + b \frac{\theta^2}{6n}$

$\log (1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$

Fixed $y$

Fixed $\theta$

Answers

Omit to $n^{-1}$