Modelling $\theta \rightarrow y$

Inference

\[ f(y; \theta) \rightleftharpoons L(\theta) = c f(y^*; \theta) \]

p-value $p(\theta) = F(y^*; \theta)$

Vector $y^*$... law; "left of"?

Vector valued $\theta$

Ex: $N(\mu, \sigma^2)$

Ex: $y = X\beta + \varepsilon$

Log. likelihood

\[ L = \prod_{i=1}^{n} \operatorname{d} \text{ate} \]

\[ L = c f(y^*; \theta) = \frac{1}{\theta(y)} \]

"Most at data" function

\[ \hat{\theta} \]

Good...
Ex: $y_1, ..., y_n \sim N(\mu, \sigma^2)$

$$f(y_i; \mu, \sigma^2) = \frac{1}{(2\pi\sigma^2)^{\frac{n}{2}}} \exp \left\{ -\frac{1}{2\sigma^2} \left[ \sum (y_i - \bar{y})^2 + n(\bar{y} - \mu)^2 \right] \right\}$$

$$L(\mu, \sigma^2) = \frac{\Gamma(n/2)}{(\sigma^2)^{n/2}} \exp \left\{ -\frac{n}{2\sigma^2} (\bar{y} - \mu)^2 \right\}$$

$$l(\mu, \sigma^2) = -\frac{n}{2} \log \sigma^2 - \frac{n}{2\sigma^2} (\bar{y} - \mu)^2 - \frac{1}{2\sigma^2} \sum (y_i - \bar{y})^2$$

$$\Rightarrow \ell_\mu = \frac{n}{\sigma^2} (\bar{y} - \mu)$$

$$\ell_\sigma^2 = -\frac{n}{2} \log \sigma^2 + \frac{S^2 + n(\bar{y} - \mu)^2}{2\sigma^4}$$

MLE = $\hat{\Theta}$: Calculus (IF) $\ell_\Theta = 0$, $\ell_\Theta = 0$:

$$\mu = \bar{y}, \quad \sigma^2 = \frac{\sum (y_i - \bar{y})^2}{n-1} = \frac{n-1}{n} \sigma_E^2$$

$$\hat{\sigma}^2 = \frac{\sum (y_i - \bar{y})^2}{n} = MS\ Res$$
\[
\ell_\mu = \frac{n}{\sigma^2} (\bar{Y} - \mu) \\
\ell_{\sigma^2} = -\frac{n}{2\sigma^2} + \frac{S^2 + n(\bar{Y} - \mu)^2}{2\sigma^2 + (r \sigma^2)} \\
\ell_\mu = \frac{n}{\sigma^2} \\
\ell_{\sigma^2} = \frac{n}{\sigma^2} (\bar{Y} - \mu) \\
\ell_{\sigma^2} = \frac{n}{2\sigma^2 - 2}
\]
\[\ell_\mu = \frac{n}{\sigma^2} \ln \left(\frac{\sigma}{n} \right)\]

\[\ell_\sigma^2 = -\frac{n}{2\sigma^2} + \frac{5^2 + n(\bar{y} - \mu)^2}{2\sigma^4} \ln \sigma^2\]

- \[\ell_\mu = \frac{n}{\sigma^2}\]

- \[\ell_\sigma^2 = \frac{n}{\sigma^2} \ln \left(\frac{\sigma}{n} \right)\]

- \[\ell_\sigma^2 = \frac{n}{2\sigma^2} + 2\frac{1}{\sigma^6} \frac{\sum(y_i - \mu)^2}{2}\]

- \[\ell_\sigma^2 = -\frac{n}{2\sigma^2} + 2n\sigma^2 \frac{\sum(y_i - \mu)^2}{2\sigma^2} = \frac{n}{2\sigma^4}\]

- \[\nabla \ell_\mu = \frac{n}{\sigma^2} \frac{1}{n}\]

- \[\nabla \ell_\sigma^2 = 0\]

- \[\nabla^2 \ell_\sigma^2 = \frac{n}{2\sigma^4}\]

- \[\lim_{\sigma \to 0^+} \ell_\sigma^2 = -\infty\]

- \[\ell_\sigma^2 \text{ neg Hessian at } \hat{\sigma}\]

- \[\frac{d}{d\sigma^2}(\ell_\sigma^2)^{-1} = (-\sigma^2)^{-2}\]

- \[\text{Observed info... Neg Hess at } \hat{\Theta}\]

- \[J = \begin{pmatrix} \frac{n}{2\sigma^4} & 0 \\ 0 & \frac{n}{2\sigma^4} \end{pmatrix}\]

- \[\text{Important!}\]
\[-l_{\mu} = \frac{n}{\sigma^2} \quad \Rightarrow \quad -\ell_{\mu}(\theta) \]

\[-l_{\mu \mu} = \frac{n}{\sigma^2} \mathbf{1} \quad \Rightarrow \quad \ell_{\mu \mu}(\theta) \]

\[-l_{\mu \sigma^2} = \frac{n}{\sigma^2} (\mathbf{1} - \mathbf{1}) \quad \Rightarrow \quad l_{\mu \sigma^2}(\theta) \]

\[-l_{\sigma^2} = -\frac{n}{2\sigma^4} + \frac{1}{\sigma^6} \sum (y_i - \mu)^2 \quad \Rightarrow \quad l_{\sigma^2}(\theta) \]

\[\text{Observed info} \quad \Rightarrow \quad \text{Neg Hess at } \hat{\theta} \]

\[\hat{\mu} = \frac{n}{\sigma^2} \]

\[\hat{\sigma^2} = \frac{n}{2\sigma^4} \]

\[\hat{\mu \sigma^2} = 0 \]

\[\hat{\mu \mu} = \frac{n}{\sigma^2} \]

\[\hat{\mu \sigma}^2 \quad \Rightarrow \quad \text{depends on } y \]

\[\hat{i}_{\mu}(\theta) = E[-l_{\mu}(\theta); \theta] = \frac{n}{\sigma^2} \]

\[\hat{i}_{\sigma^2}(\theta) = E[-l_{\sigma^2}(\theta); \theta] = 0 \]

\[\hat{i}_{\sigma^2}(\theta) = E[-l_{\sigma^2}(\theta); \theta] = \frac{n}{2\sigma^4} \]

\[\hat{l}_{\sigma^2}(\theta) = \begin{pmatrix} \frac{n}{\sigma^2} & 0 \\ 0 & \frac{n}{2\sigma^4} \end{pmatrix} \]

\[\hat{l}_{\theta}(\theta) = \begin{pmatrix} \frac{n}{\sigma^2} & 0 \\ 0 & \frac{n}{2\sigma^4} \end{pmatrix} \]
Overview

$f(y;\theta)$ Model

$p(\theta; y)$ Lik

$l_q(\theta; y)$ Grad Slope

$-\frac{\partial}{\partial \theta} l(\theta; y)$ - Hessian

$\text{NB} \quad E\{l_0(\theta; y); \theta\} = 0$  
$V\{l_0(\theta; y); \theta\} = -E\{l_0(\theta; y); \theta\}$

Barrett Eqs.

Important to ensure Lik/Bayes statistics

and Saw for N1 example