Bayes I (1763) of Laplace 1812, 1820 Gauss 1777-1855

Set-up
\[ y|\theta \sim f(y; \theta) \mathrm{d}\theta \] Model (1)
\[ \theta \sim \pi(\theta) \]

prior
source

Theory
As \( f(\pi(\theta)) \) is near
\[ y|\theta \sim f(y; \theta) \mathrm{d}\theta \]
\[ \theta \sim \pi(\theta) \mathrm{d}\theta \]

\((\theta, y) \sim \pi(\theta)f(y; \theta) \mathrm{d}\theta \mathrm{d}y\)

Data \( y^0 \): Obs something re variable \((y, \theta)\)
\[ y = y^0 \]

Standard prob: Condit data for \( \theta | y = y^0 \)

Dec.: No imperative say that you MUST use

If you decide to

Available \( \theta | y^0 \sim \text{Joint}(\theta, y^0) \) norm

\( y^0 \): Condit is just a section of \( \text{Joint} \)
$y^\circ \in C_T(\theta) L(\theta; y^\circ) \sim \text{Joint on } y^\circ \text{ section}$

Easy: Use $L^o(\theta)$

"Add" scaling weight fn

Posterior $\rightarrow$ Probabilities

(say freq could not call conf. a prob) own turf

$C = \frac{1}{\text{Marginal for } y \text{ at } y^\circ}$

Condition $= \frac{\text{Joint}}{\text{Marginal pdf at } y^\circ}$

$\frac{P(A|c)}{P(c)}$
Example: $\pi(\theta)$ is REAL (Objective)

$\theta$ Int. char $\sim N(100; 10)$

Measure: Instrument $y \sim N(\theta; 5)$ $L(\theta; 120) = ce^{-\frac{1}{2} \frac{(120 - \theta)^2}{5}}$

Person $y = y^0 = 120$ --- But $\pi(\theta | y^0)$

If you combine $\pi(\theta | y^0) =$ Posterior for $\theta$ when $y = 120$

$= \pi(\theta) L(\theta)$

- Shrinking
- Prejudice

Do you buy it??

false: & false or false-ve
sensitivity: specificity
Ex: $\theta \sim N(100, 10)$  $y \sim N(\theta, 5)$  $\theta | y = 120$

Prior = $p(\theta) = \frac{1}{\sqrt{2\pi \sigma^2}} \exp \left\{ -\frac{1}{2\sigma^2} (\theta - 100)^2 \right\}$

Lik = $L(\theta) = c \exp \left\{ -\frac{1}{2\sigma^2} (\theta - 120)^2 \right\}$

Posterior = $C(y^o) \exp \left\{ -\frac{1}{2} \left( \frac{1}{\sigma^2} + \frac{1}{\sigma'^2} \right) \theta^2 + \left( \frac{100}{\sigma^2} + \frac{120}{\sigma'^2} \right) \theta + \ldots \right\}$

$= \frac{1}{\sqrt{2\pi}} \exp \left\{ -\frac{1}{2} \frac{1}{\bar{\sigma}^2} \right\}$

$\bar{\sigma}^2 = \frac{1}{10^2} + \frac{1}{5^2}$

$v(y) = \left( \frac{1}{10^2} + \frac{1}{5^2} \right)^{-1} \left( \frac{100}{10^2} + \frac{120}{5^2} \right)$

Weight by recup variance

Easy!

As check, does guy who got 120 like being chopped 1/6?
Ex2: Genetics; REAL prior (Objective) co-opted Bayes1963

Fisher

Black mouse from mating (Bb ⇒ BB Bb bb) Black Priors: Math

Get info: Mate with bb Brown

Get: 7 Black mice

Prior: θ

Mouse in Q is black

Mate with bb (Get 7 Black)

Poisson

Non

<table>
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θ = Type for mouse

Priors

L(θ)

Odds have changed