
The logic; Neyman diagram: final validation?

\[
\begin{align*}
\hat{\beta}_n & = \beta_n + t_{2.5\%} \sqrt{C_{na}} \Delta E \\
\hat{\beta}_n & = \beta_n + t_{2.5\%} \sqrt{\text{C}_{na}} \Delta E \\
t^2 & = \frac{SS_a - SS_{n-1}}{SSR/(n-n)} = F \\
\text{Assume} & \text{ for } Mon
\end{align*}
\]

The logic: Neyman diagram: final validation. §10.5 p 579

Procedure:

(a) For each \( \theta \), let \( A(\theta) \) have prob \( 1 - \alpha \) for that \( \theta \)

\[ A(\theta) \text{ = acceptance} \]

\[ P(y \in A(\theta); \theta) = 1 - \alpha = \beta \]

(b) On \( S \times \Omega \) let \( A \) be composite set

\[ A = \bigcup_{\theta \in \Omega} A(\theta) \times \{ \theta \} = \{ (y, \theta) : y \in A(\theta) \} \]

(c) For each \( y \in S \) let \( C(y) \) be \( y \)-section of \( A \)

\[ C(y) = \{ \theta : (y, \theta) \in A \} \]

\[ = \{ \theta : y \in A(\theta) \} \]

Then:

\[ P(y \in A(\theta); \theta) = 1 - \alpha = P(\theta \in C(y); \theta) \]

Prob that \( C(y) \) contains \( \theta \) is \( \beta \)

(\( \beta \) confidence)
\[ P(y \in A(\theta); \theta \in \mathcal{A}) = 1 - \alpha = \beta \approx P(\theta \in C(y); \theta) \]

In practice:

\[ \begin{align*}
1_A(y, \theta) & = 1 \quad \text{on } A \\
& = 0 \quad \text{o/w}
\end{align*} \]

Note \( 1_A(y, \theta) \) is \( \text{Bern}(\beta) \)

Note It is a pivot (with fixed distr) in \( f(y, \theta) \)

Have data \( y^0 \)

Have (in the background) a way of checking whether a data point \( y^0 \) is acceptable via some criterion \( (\text{Re } \beta) \) 95%.

Bundle acceptable \( \Rightarrow C(y^0) \)

Only need for \( y = y^0 \)

Need \( \beta \)-criterion
Simple Ex $\mu \sim N(\theta, 2.25)$

Want Conf region ... need logic

Conf lower bound!

Normal: easier

Have $\beta = 95%$

$$A(\theta) = (\theta - 1.64 \times 1.5, \infty)$$

SD = 1.5

-1.64 is 95% for $N(\theta, 1)$

$\theta - 1.64 \leq \theta_0$

Took acc. region in wrong dirn.

Hep CERN LHC

Data: Poisson ($\theta$)

Know $\theta \geq 0$ background

Wander: $\theta > \theta_0$

Papers

Sineva: Nobel

2-pred Conf
Lesson: Conf. Bds are nonstandard.

Important:

a) Get $A(\theta)$ with 95%:

$$A(\theta) = (-\infty, \theta + 2.46)$$

b) Composite

$$A = \{ A(\theta) \times \{ \theta \} \}$$

c) $y$ section $C(y)$ from $A$.

Reminder: HEPs needed a 99% conf. lower bound.

Ponder:
An example to ponder

\[ y \sim U(\theta \pm 1) \]

Examine \((y_1, y_2) \mid \theta \sim U(\theta \pm 1)\)

More

What would you do?

\[ \frac{1}{2f} \]

\( \Theta - 1 \quad \Theta \quad \Theta + 1 \)

\( \text{Unif on } [\theta \pm 1]^2 \)

\( (\Theta - 1, \Theta + 1)^2 \) Circles!