Some second order analysis for exponential models

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1 Introduction

Exponential models provide a highly flexible extension of the Normal statistical model. We examine some second order properties of an asymptotic exponential model and focus on a third derivative information that describes how nuisance information can depend on an interest parameter. Consider the exponential model with the two-dimensional canonical parameter $\varphi = (\psi, \lambda)$ and the two-dimensional canonical variable (s, t),

$$f(s, t; \psi, \lambda) = \phi(s - \psi)\phi(t - \lambda)e^{-a\psi\lambda^2/2n^{1/2}} h(s, t), \tag{1}$$

where $\phi(z)$ is the standard Normal density function and $h(s, t)$ is the underlying density. The particular likelihood third derivative term $-a\psi\lambda^2/2n^{1/2}$ has special properties on interest and the coefficient $a = \jmath_{\lambda\lambda\psi} = \ell_{\lambda\lambda\psi}$ measures from a λ information varies with respect to ψ. We will first evaluate $h(s, t)$ to second order.

As the model (1) is a probability density function for each parameter value, we have

$$\begin{aligned} 1 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(s, t; \psi, \lambda) ds dt &= e^{-a\psi\lambda^2/2n^{1/2}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \phi(s - \psi)\phi(t - \lambda)h(s, t) ds dt \\ &= e^{-a\psi\lambda^2/2n^{1/2}} E[h(s, t)], \end{aligned}$$

where the mean is calculated using the standard Normal distribution centered at (ψ, λ). We first expand the norming constant in terms of the parameter,

$$
\begin{aligned}
E[h(s,t)] &= e^{a\psi\lambda^2/2n^{1/2}} \\
&= 1 + a\psi\lambda^2/2n^{1/2} + a^2\psi^2\lambda^4/8n,
\end{aligned}
$$

to the third order. Then by using the fact that $E(y^2-1)=\theta^2$ and $E(y^4-6y^2+3)=\theta^4$ where y is location Normal with centre θ, we can deduce the expansion of $h(s,t)$:

$$h(s,t) = 1 + as(t^2-1)/2n^{1/2} + a^2(s^2-1)(t^4-6t^2+3)/8n.$$

By substituting the expansion of $e^{-a\psi\lambda^2/2n^{1/2}}$ and $h(s,t)$ both to the third order, we obtain a more detailed form of (1).

$$
\begin{aligned}
f(s,t;\psi,\lambda) &= \phi(s-\psi)\phi(t-\lambda)\{1 - a\psi\lambda^2/2n^{1/2} + a^2\psi^2\lambda^4/8n\} \cdot \\
&\quad \cdot\{1 + as(t^2-1)/2n^{1/2} + a^2(s^2-1)(t^4-6t^2+3)/8n\}. \qquad (2)
\end{aligned}
$$

Expanding (2) we obtain

$$
\begin{aligned}
f(s,t;\psi,\lambda) &= \phi(s-\psi)\phi(t-\lambda)\{1 + as(t^2-1)/2n^{1/2} + a^2(s^2-1)(t^4-6t^2+3)/8n \\
&\quad -a\psi\lambda^2/2n^{1/2} - a^2 s\psi\lambda^2(t^2-1)/4n + a^2\psi^2\lambda^4/8n\} \\
&= \phi(s-\psi)\phi(t-\lambda)\{1 + \frac{a}{2n^{1/2}}[s(t^2-1) - \psi\lambda^2] \\
&\quad + \frac{a^2}{8n}[(s^2-1)(t^4-6t^2+3) - 2s\psi\lambda^2(t^2-1) + \psi^2\lambda^4]\}. \qquad (3)
\end{aligned}
$$

We can then verify that the expression for $f(s,t;\psi,\lambda)$ is indeed a density function:

$$
\begin{aligned}
\int\int f(s,t;\psi,\lambda)dsdt &= \int\int \phi(s-\psi)\phi(t-\lambda)dsdt \\
&\quad + \int\int \phi(s-\psi)\phi(t-\lambda)a[s(t^2-1)-\psi\lambda^2]/2n^{1/2}dsdt
\end{aligned}
$$

$$
\begin{aligned}
& + \int\int \phi(s-\psi)\phi(t-\lambda)\frac{a^2}{8n}[(s^2-1)(t^4-6t^2+3) \\
& -2s\psi\lambda^2(t^2-1)+\psi^2\lambda^4]dsdt \\
= \ & 1+\frac{a}{2n^{1/2}}[\psi\lambda^2-\psi\lambda^2]+\frac{a^2}{8n}[\psi^2\lambda^4-2\psi^2\lambda^2\lambda^2+\psi^2\lambda^4] \\
= \ & 1.
\end{aligned}
$$

We then expand the right side of (3) for easier checking with a saddlepoint version later:

$$
\begin{aligned}
f(s,t;\psi,\lambda) \ = \ & \phi(s-\psi)\phi(t-\lambda)\{1+\frac{a}{2n^{1/2}}[s(t^2-1)-\psi\lambda^2] \\
& +\frac{a^2}{8n}[s^2t^4-6s^2t^2+3s^2-2s\psi\lambda^2t^2+2s\psi\lambda^2-t^4+6t^2-3+\psi^2\lambda^4]\}.
\end{aligned}
$$

Alternatively, $f(s,t;\psi,\lambda)$ can also be rewritten as

$$
\begin{aligned}
f(s,t;\psi,\lambda) \ = \ & \phi(s-\psi)\phi(t-\lambda)\{1-a\psi\lambda^2/2n^{1/2}+a^2\psi^2\lambda^4/8n\}\cdot \\
& \cdot\{1+ast^2/2n^{1/2}-as/2n^{1/2}+\frac{a^2}{8n}(s^2t^4-6s^2t^2+3s^2-t^4+6t^2-3)\}, \quad (4)
\end{aligned}
$$

which keeps the parameter pieces in one bracket and the data pieces in another.

2 Saddlepoint reexpression of the exponential model

The expression (2) can also be derived using saddlepoint approach. We write out the exponential model in more detail,

$$
f(s,t;\psi,\lambda)=\frac{1}{2\pi}\exp\left\{-\frac{(s-\psi)^2}{2}-\frac{(t-\lambda)^2}{2}-a\psi\lambda^2/2n^{1/2}\right\}h(s,t). \quad (5)
$$

The log-likelihood function for (5) is

$$
\ell(\psi,\lambda;s,t)=-\frac{\psi^2}{2}-\frac{\lambda^2}{2}+\psi s+\lambda t-a\psi\lambda^2/2n^{1/2}.
$$

First we find the maximum likelihood estimates for ψ and λ, then the information $\hat{\jmath} = \hat{\jmath}(s,t)$, and then the saddlepoint reexpression.

Some simple derivatives give the score equations,

$$\begin{aligned} \ell_\psi &= s - \psi - a\lambda^2/2n^{1/2}, \\ \ell_\lambda &= t - \lambda - a\psi\lambda/n^{1/2}, \end{aligned}$$

and then give the information quantiles,

$$\begin{aligned} \ell_{\psi\psi} &= -1 & \jmath_{\psi\psi} &= 1 \\ \ell_{\psi\lambda} &= -a\lambda/n^{1/2} & \jmath_{\psi\lambda} &= a\lambda/n^{1/2} \\ \ell_{\lambda\lambda} &= -1 - a\psi/n^{1/2} & \jmath_{\lambda\lambda} &= 1 + a\psi/n^{1/2} \end{aligned}$$

Solving the score equation for the maximum likelihood estimates $\hat{\psi}$, $\hat{\lambda}$ to the third order gives

$$\begin{aligned} \hat{\psi} = s - a\hat{\lambda}^2/2n^{1/2} &= s - (t - a\hat{\psi}\hat{\lambda}/n^{1/2})^2/2n^{1/2} \\ &= s - a(t^2 - 2at\hat{\psi}\hat{\lambda}/n^{1/2})/2n^{1/2} \\ &= s - at^2/2n^{1/2} + a^2st^2/n \\ \hat{\lambda} = t - a\hat{\psi}\hat{\lambda}/n^{1/2} &= t - a(s - a\hat{\lambda}^2/2n^{1/2})(t - a\hat{\psi}\hat{\lambda}/n^{1/2})/n^{1/2} \\ &= t - ast/n^{1/2} + a^2s^2t/n + a^2t^3/2n, \end{aligned}$$

by iteratively re-substituting and by replacing $\hat{\lambda}$ by t and $\hat{\psi}$ by s in the second order n^{-1} terms.

The information matrix is

$$\hat{\jmath}_{\varphi\varphi} = \hat{\jmath}_{\varphi\varphi}(s,t) = \begin{pmatrix} 1 & a\hat{\lambda}/n^{1/2} \\ a\hat{\lambda}/n^{1/2} & 1 + a\hat{\psi}/n^{1/2} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & a(t - ast/n^{1/2})/n^{1/2} \\ a(t - ast/n^{1/2})/n^{1/2} & 1 + a(s - at^2/2n^{1/2})/n^{1/2} \end{pmatrix}.$$

The norm of $\hat{\jmath}_{\varphi\varphi}$ is

$$\begin{aligned} |\hat{\jmath}_{\varphi\varphi}| &= 1 + a(s - at^2/2n^{1/2})/n^{1/2} - a^2t^2/n \\ &= 1 + as/n^{1/2} - a^2t^2/2n - a^2t^2/n \\ &= 1 + as/n^{1/2} - \frac{3}{2}a^2t^2/n; \end{aligned}$$

and thus

$$|\hat{\jmath}_{\varphi\varphi}|^{-1/2} = 1 - as/2n^{1/2} + \frac{3}{4}a^2t^2/n + \frac{3}{8}a^2s^2/n$$

using $(1+x)^{-1/2} = 1 - \frac{1}{2}x + \frac{3}{8}x^2$ to the third order.

The saddlepoint approximation of the model is

$$\begin{aligned} f(s,t;\psi,\lambda)dsdt &= \frac{e^{k/n}}{2\pi}\exp\{\ell - \hat{\ell}\}|\hat{\jmath}_{\varphi\varphi}(s,t)|^{-1/2} \cdot dsdt \qquad (6) \\ &= \phi(s-\psi)\phi(t-\lambda)e^{-a\psi\lambda^2/2n^{1/2}}e^{k/n}e^{-\hat{\ell}}|\hat{\jmath}_{\varphi\varphi}|^{-1/2}dsdt \end{aligned}$$

where $\ell - \hat{\ell} = \ell(\psi,\lambda;s,t) - \ell(\hat{\psi},\hat{\lambda};s,t)$ and $\ell = -(s-\psi)^2/2 - (t-\lambda)^2/2 - a\psi\lambda^2/2n^{1/2}$, and

$$\begin{aligned} \hat{\ell} &= -(s-\hat{\psi})^2/2 - (t-\hat{\lambda})^2/2 - a\hat{\psi}\hat{\lambda}^2/2n^{1/2} \\ &= -\frac{1}{2}(at^2/2n^{1/2})^2 - \frac{1}{2}(ast/n^{1/2})^2 - a(s - at^2/2n^{1/2})(t - ast/n^{1/2})^2/2n^{1/2} \\ &= -\frac{1}{2}a^2t^4/4n - \frac{1}{2}a^2s^2t^2/n - ast^2/2n^{1/2} + a^2t^4/4n + 2a^2s^2t^2/2n \\ &= -ast^2/2n^{1/2} + a^2t^4/8n + a^2s^2t^2/2n. \end{aligned}$$

Thus

$$e^{k/n}e^{-\hat{\ell}}|\hat{\jmath}_{\varphi\varphi}|^{-1/2} = e^{k/n}\exp\left\{\frac{ast^2}{2n^{1/2}} - \frac{a^2t^4}{8n} - \frac{a^2s^2t^2}{2n}\right\}\left\{1 - \frac{as}{2n^{1/2}} + \frac{3a^2t^2}{4n} + \frac{3a^2s^2}{8n}\right\}$$

$$
\begin{aligned}
&= e^{k/n}\left\{1 + ast^2/2n^{1/2} - a^2t^4/8n - \frac{a^2s^2t^2}{2n} + \frac{a^2s^2t^4}{8n}\right\}\left\{1 - \frac{as}{2n^{1/2}} + \frac{3a^2t^2}{4n} + \frac{3a^2s^2}{8n}\right\} \\
&= e^{k/n}\left\{1 + \frac{ast^2}{2n^{1/2}} - \frac{a^2s^2t^2}{4n} - \frac{a^2t^4}{8n} - \frac{a^2s^2t^2}{2n} + \frac{a^2s^2t^4}{8n} - \frac{as}{2n^{1/2}} + \frac{3a^2t^2}{4n} + \frac{3a^2s^2}{8n}\right\} \\
&= e^{k/n}\left\{1 + ast^2/2n^{1/2} - as/2n^{1/2} + \frac{a^2}{8n}(s^2t^4 - 6s^2t^2 + 3s^2 - t^4 + 6t^2)\right\} \\
&= e^{k/n}\left\{1 + ast^2/2n^{1/2} - as/2n^{1/2} + \frac{a^2}{8n}(s^2t^4 - 6s^2t^2 + 3s^2 - t^4 + 6t^2)\right\} \\
&= 1 + ast^2/2n^{1/2} - as/2n^{1/2} + \frac{a^2}{8n}(s^2t^4 - 6s^2t^2 + 3s^2 - t^4 + 6t^2 - 3),
\end{aligned}
$$

from which we deduce $k/n = 3a^2/8n$ using the connection with (4).

3 Conditional analysis of the exponential model

Consider the conditional analysis for $s|t$ with the exponential model (2); we have

$$
f(s|t;\psi) = c\phi(s-\psi)\{1 + as(t^2-1)/2n^{1/2} + a^2(s^2-1)(t^4-6t^2+3)/8n\}, \tag{7}
$$

and c is determined by

$$
\int f(s|t;\psi)ds = c\{1 + a\psi(t^2-1)/2n^{1/2} + a^2\psi^2(t^4-6t^2+3)/8n\} = 1,
$$

which leads to

$$
\begin{aligned}
c &= \{1 + a\psi(t^2-1)/2n^{1/2} + a^2\psi^2(t^4-6t^2+3)/8n\}^{-1} \\
&= 1 - a\psi(t^2-1)/2n^{1/2} - a^2\psi^2(t^4-6t^2+3)/8n + a^2\psi^2(t^4-2t^2+1)/4n \\
&= 1 - a\psi(t^2-1)/2n^{1/2} + a^2\psi^2(t^4+2t^2-1)/8n \\
&= \exp\{-a\psi(t^2-1)/2n^{1/2} + a^2\psi^2(t^4+2t^2-1)/8n - a^2\psi^2(t^2-1)^2/8n\} \\
&= \exp\{-a\psi(t^2-1)/2n^{1/2} + a^2\psi^2(4t^2-2)/8n\}
\end{aligned}
$$

where $(1+x)^{-1} = 1 - x + x^2$ and $\log(1+x) = x - x^2/2$ to the third order. The conditional distribution (7) becomes

$$\begin{aligned} f(s|t;\psi) &= \phi(s-\psi)\{1 + as(t^2-1)/2n^{1/2} + a^2(s^2-1)(t^4-6t^2+3)/8n\} \cdot \\ &\quad \{1 - a\psi(t^2-1)/2n^{1/2} + a^2\psi^2(t^4+2t^2-1)/8n\} \\ &= \phi(s-\psi)\{1 + as(t^2-1)/2n^{1/2} + a^2(s^2-1)(t^4-6t^2+3)/8n\} \cdot \\ &\quad \exp\{-a\psi(t^2-1)/2n^{1/2} + a^2\psi^2(4t^2-2)/8n\} \end{aligned}$$

which is a probability density function by direct integration.

4 Marginal third order analysis of the exponential model

We would like to determine the marginal model for assessing ψ using third order method. Let $S(s,t)$ be a third order ancillary with respect to λ for a given ψ. First we find the saddlepoint version of the probability density function for S:

$$\begin{aligned} f(S;\psi)dS &= \frac{e^{k/n}}{(2\pi)^{1/2}} \frac{L(\psi, \hat{\lambda}_\psi; s, 0)}{L(\hat{\psi}, \hat{\lambda}; s, 0)} |\jmath^{\psi\psi}(s,0)|^{1/2} \cdot dS \\ &= c\exp\{\ell(\psi, \hat{\lambda}_\psi; s, 0) - \ell(\hat{\psi}, \hat{\lambda}; s, 0)\} |\jmath^{\psi\psi}(s,0)|^{1/2} \cdot dS. \end{aligned} \tag{8}$$

Looking at the components piece by piece, we first obtain

$$\ell(\hat{\psi}, \hat{\lambda}; s, 0) = -\frac{(s-\hat{\psi})^2}{2} - \frac{(t-\hat{\lambda})^2}{2} - a\hat{\psi}\hat{\lambda}^2/2n^{1/2} = 0,$$

which uses the maximum likelihood estimates from Section 2. Also, we have the score equation $\partial\ell/\partial\lambda = t - \lambda - a\psi\lambda/n^{1/2}$, which leads to $\hat{\lambda}_\psi = t - a\psi\lambda/n^{1/2} = t - a\psi t/n^{1/2} + a^2\psi^2 t/n$. Then when $t = 0$ we obtain $\hat{\lambda}_\psi = 0$, which in turn gives

$$\ell(\psi, \hat{\lambda}_\psi; s, 0) = -\frac{(s-\psi)^2}{2}.$$

The information matrix is

$$
\begin{aligned}
\jmath_{\varphi\varphi}(s,0) &= \begin{pmatrix} 1 & a(t-ast/n^{1/2})/n^{1/2} \\ a(t-ast/n^{1/2})/n^{1/2} & 1+a(s-at^2/2n^{1/2})/n^{1/2} \end{pmatrix}(s,0) \\
&= \begin{pmatrix} 1 & 0 \\ 0 & 1+as/n^{1/2} \end{pmatrix},
\end{aligned}
$$

thus $|\jmath_{\varphi\varphi}| = 1 + as/n^{1/2}$ and

$$|\jmath^{\psi\psi}(s,0)| = |\jmath_{\varphi\varphi}|/\jmath_{\lambda\lambda} = \frac{1+as/n^{1/2}}{1+as/n^{1/2}} = 1.$$

Then from (8) we have

$$f(S;\psi) = ce^{-\frac{1}{2}(S-\psi)^2} \cdot 1 = \phi(S-\psi).$$

where $\phi(z)$ is the standard Normal density function.

5 Joint distribution for (S,t).

We now examine the joint distribution $f(S,t;\psi,\lambda)$ given by (2). For this we try $S = s - a(t^2-1)/2n^{1/2}$ and substitute in the following

$$
\begin{aligned}
\phi(s-\psi) &= \phi(S + a(t^2-1)/2n^{1/2} - \psi) \\
&= \frac{1}{\sqrt{2\pi}} e^{-(S-\psi)^2/2-(S-\psi)a(t^2-1)/2n^{1/2}} \\
&= \phi(S-\psi)\{1-(S-\psi)a(t^2-1)/2n^{1/2}\},
\end{aligned}
$$

where ϕ is the usual standard Normal density function. Note the Jacobian from (s,t) to (S,t) is unity. Then the substitution of $s = S + a(t^2-1)/2n^{1/2}$ into (2) gives

$$
\begin{aligned}
f(S,t;\psi,\lambda) &= \phi(S-\psi)\phi(t-\lambda)\exp\{-a\psi\lambda^2/2n^{1/2} - (S-\psi)a(t^2-1)/2n^{1/2} + aS(t^2-1)2n^{1/2}\} \\
&= \phi(S-\psi)\phi(t-\lambda)\exp\{-a\psi[\lambda^2-(t^2-1)]/2n^{1/2}\}.
\end{aligned}
$$

Now will move to the third order. Similar to the second order case,

$$
\begin{aligned}
\phi(s-\psi) &= \phi(S-\psi+a(t^2-1)/2n^{1/2}) \\
&= \frac{1}{\sqrt{2\pi}}\exp\{-(S-\psi)^2/2-(S-\psi)a(t^2-1)/2n^{1/2}-a^2(t^2-1)^2/8n\} \\
&= \phi(S-\psi)\{1-(S-\psi)a(t^2-1)/2n^{1/2}-a^2(t^2-1)^2/8n+(S-\psi)^2a^2(t^2-1)^2/8n\} \\
&= \phi(S-\psi)\{1-(S-\psi)a(t^2-1)/2n^{1/2}+[(S-\psi)^2-1]a^2(t^2-1)^2/8n\}.
\end{aligned}
$$

Making the same substitution to (2) gives

$$
\begin{aligned}
& \{1+as(t^2-1)/2n^{1/2}+a^2(s^2-1)(t^4-6t^2+3)/8n\} \\
= \ & 1+aS(t^2-1)/2n^{1/2}+a^2(t^2-1)^2/4n+a^2(S^2-1)(t^4-6t^2+3)/8n \\
= \ & 1+aS(t^2-1)/2n^{1/2}+a^2S^2(t^4-6t^2+3)/8n+\{2a^2(t^4-2t^2+1)-a^2(t^4-6t^2+3)\}/8n \\
= \ & 1+aS(t^2-1)/2n^{1/2}+a^2S^2(t^4-6t^2+3)/8n+a^2(t^4+2t^2-1)/8n
\end{aligned}
$$

Some simplification gives the final joint density function $f(S,t;\psi,\lambda)$,

$$
\begin{aligned}
f(S,t;\psi,\lambda) &= \phi(S-\psi)\phi(t-\lambda)\exp\{-a\psi\lambda^2/2n^{1/2}\}\cdot \\
&\quad \cdot\{1-(S-\psi)a(t^2-1)/2n^{1/2}+[(S-\psi)^2-1]a^2(t^2-1)^2/8n\}\cdot \\
&\quad \cdot\{1+aS(t^2-1)/2n^{1/2}+a^2[S^2(t^4-6t^2+3)+(t^4+2t^2-1)]/8n\} \\
&= \phi(S-\psi)\phi(t-\lambda)\exp\{-a\psi(\lambda^2-t^2+1)/2n^{1/2}\}\cdot \\
&\quad \cdot\{1-a^2S(S-\psi)(t^2-1)^2/4n+(S-\psi)^2a^2(t^2-1)^2/8n-a^2(t^2-1)^2/8n \\
&\quad -a^2\psi^2(t^2-1)^2/8n+a^2S^2(t^4-6t^2+3)/8n+a^2(t^4+2t^2-1)/8n\} \\
&= \phi(S-\psi)\phi(t-\lambda)\exp\{-a\psi(\lambda^2-t^2+1)/2n^{1/2}\}\cdot \\
&\quad \cdot\{1-a^2S^2(t^2-1)^2/4n+a^2S\psi(t^2-1)^2/4n+S^2a^2(t^2-1)^2/8n-2S\psi a^2(t^2-1)^2/8n \\
&\quad +a^2\psi^2(t^2-1)^2/8n-a^2(t^2-1)^2/8n \\
&\quad -a^2\psi^2(t^2-1)^2/8n+a^2S^2(t^4-6t^2+3)/8n+a^2(t^4+2t^2-1)/8n\}
\end{aligned}
$$

$$
\begin{aligned}
&= \phi(S-\psi)\phi(t-\lambda)\exp\{-a\psi(\lambda^2-t^2+1)/2n^{1/2}\}\cdot \\
&\quad \cdot\{1+\frac{a^2}{8n}[-S^2(t^2-1)^2-(t^2-1)^2+S^2(t^4-6t^2+3)+(t^4+2t^2-1)]\} \\
&= \phi(S-\psi)\phi(t-\lambda)\exp\{-a\psi(\lambda^2-t^2+1)/2n^{1/2}\}\{1-a^2(S^2-1)(4t^2-2)/8n\}.
\end{aligned}
$$

This joint density indeed integrates to 1,

$$
\begin{aligned}
& \int\int \phi(S-\psi)\phi(t-\lambda)\exp\{a\psi(t^2-1)/2n^{1/2}\}\{1-a^2(S^2-1)(4t^2-2)/8n\}dSdt \\
= & \int \phi(t-\lambda)\{1+a\psi(t^2-1)/2n^{1/2}+a^2\psi^2(t^2-1)^2/8n\}\{1-a^2\psi^2(4t^2-2)/8n\}dt \\
= & \int \phi(t-\lambda)\{1+a\psi(t^2-1)/2n^{1/2}+a^2\psi^2[t^4-2t^2+1-4t^2+2]/8n\}dt \\
= & \; 1+a\psi\lambda^2/2n^{1/2}+a^2\psi^2\lambda^4/8n \\
= & \; \exp\{a\psi\lambda^2/2n^{1/2}\}.
\end{aligned}
$$

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