The most widely applied criteria are for chains that "drift" toward either \(0, \ldots, N\) or infinity. For chains that may have a mixture of drifts, toward zero for some states and toward infinity for others, criteria for positive recurrence are given in refs. 8 and 9.

Recent work on Markov chain theory has shown that the classical positive recurrence and recurrence definitions extend to very general state spaces [10], and of particular practical importance is the extension to real-valued or Euclidean-space-valued chains. The recurrence criteria described above extend to much more general contexts, with the set \(\{0, \ldots, N\}\) typically replaced by a compact set.

Again the identity function is the most common test function when the state space is \((0, \infty)\). Multidimensional chains are intrinsically more difficult to handle. One extension of Foster's results is in ref. 7, illustrating the use of a multidimensional criterion. The more sophisticated use of a one-dimensional criterion in ref. 1 for vector autoregressive time series* processes shows that the use of a quadratic form as a test function recovers known conditions for second-order stationarity from a Markov chain approach.

References


(MARKOV PROCESSES
QUEUEING THEORY
RANDOM WALK
RETURN STATE)

R. L. TWEEDIE

RECURRENT EVENTS  See RENEWAL THEORY

RECURRENT STATE  See MARKOV PROCESSES

REduced MODEL

The standard basis for statistical inference involves a statistical model and data; see, for example, INFERENCE, STATISTICAL: I, II. The model might be: \(y_1, \ldots, y_n\), independent, identically distributed (i.i.d.) \((\mu, \sigma^2)\) with \((\mu, \sigma^2)\) in \(R \times R^+\); and the data might be \(y_1^0, \ldots, y_n^0\). A reduced model obtained from this is: \(\bar{y}\) is normal \((\mu, \sigma^2/n)\) and inde-

---
pendently $\sum(y - \bar{y})^2$ is $\sigma^2 \chi^2$ where $\chi^2$ is chi-square $(n - 1)$; the relevant corresponding data are $(\bar{y}, \sum(y^2 - \bar{y}))$. This reduced model applies on $R \times R^+$ and is a major simplification of the original model on the $n$-dimensional sample space $R^n$.

The reduction to the reduced model in the preceding example can be based on the sufficiency principle, or on the invariance principle* (see also rotation group), or on the conditionality principle (see also ancillary statistics), or on the weak likelihood principle. Principles of statistical inference as just indicated quite commonly lead to a reduced or simplified statistical model.

Reduced models can, however, arise deductively without recourse to principles of inference. As a first example suppose that $\theta$ has occurred as a realized value from a prior density $p(\theta)$ and that $y$ comes from the statistical model $f(y | \theta)$. The initial model is then $p(\theta)f(y | \theta)$ and the observed datum is, say, $y^0$. The use of probability as part of the modeling process then predicates the reduced model $cp(\theta)f(y | \theta)$, the conditional density for $\theta$ given $y^0$. For a related discussion, see BAYESIAN INFERENCE.

As a second example, consider the error or structural model* $y_1 = \theta + e_1, \ldots, y_n = \theta + e_n$, where $e_1, \ldots, e_n$ is a sample from the normal $(0, \sigma_0^2$) [or from some given density $f(e)$] and $\theta$ is the related data be, say, $y_1^0, \ldots, y_n^0$. The data allow the calculation of $(e_1 - \bar{e}, \ldots, e_n - \bar{e}) = (y_1^0 - y^0, \ldots, y_n^0 - y^0)$; thus all but one degree of freedom for the $e$'s is known. As in the preceding example, this predicates the conditional model: $\bar{y} = \theta + \bar{e}$, where $\bar{e}$ has the conditional distribution $cf(\bar{e} - \theta + y^0 - \bar{y}^0)f(\bar{e} - \theta + y^0 - \bar{y}^0)$ and the related datum is $\bar{y}^0$. This is a reduction from a model on $R^n$ to a reduced model on $R^1$. See STRUCTURAL INERENCE for various generalizations.

**Bibliography**


**(ANCILLARY STATISTICS**

[STRUCTURAL INFERENCE
SUFFICIENT STATISTICS]

D. A. S. Fraser

**REDDUCIBLE CHAIN See MARKOV PROCESSES**

**REDUCTION OF DATA**

Reducing observed data to summary figures is a central part of statistics. Fisher [4, p. 1] referred to the study of methods of the reduction of data as being one of the three main aspects of statistics. (The other two are the study of populations and the study of variation.)

One use of the term is in reducing the dimensions of multivariate data, as in factor analysis* or correlational analyses more generally (e.g., Simon [5]). But a more recent use stems from the fact that it is not unusual for statistical workers to apply analysis techniques to their data without ever having "looked at the data." For example,

In the analysis of variance*, they may report $F$-ratios and significance levels, but not the mean values.

In factor analysis*, they may report the factor loadings and amounts of variance accounted for, but not the observed correlations.

As a reaction, there has been renewed emphasis on data analysis. In Tukey's exploratory data analysis* (EDA) the focus is on exploration (i.e., finding patterns and exceptions in data that are new to the analyst). Data reduction is a more general term used for a boiling down of any data, including repetitive kinds such as occur in information systems. A particular aim is to facilitate the comparison of different data sets, so as to lead to the empirical generalizations and lawlike relationships* of ordinary science.

The term "data reduction" has also become associated with a narrow range of rules or procedures designed to help the analyst to see and to communicate the struc-