# Interaction effects

A manufacturer was finding unwanted differences in the torque values of a locknut that it made. Torque is the work (i.e. force times distance) required to tighten the nut. The manufacturer conducted an experiment to determine what factors affected the torque values. The type of plating process was isolated as the most probable factor. Another factor is the test medium that is, the whether the locknut is threaded onto a bolt or a mandrel. Therefore, the two experimental factors were type of plating, whose three levels were no plating, cadmium plated and phosphate plated, and test medium whose levels were mandrel and bolt. The torque values are given below. First 10 rows are for the mandrel and last ten correspond to bolt.

```r
bolt <- read.table("LinMod-1/Design/bolt.dat", header=T)
bolt
```

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10</td>
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<td>32</td>
<td>M</td>
</tr>
<tr>
<td>2</td>
<td>13</td>
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<td>22</td>
<td>M</td>
</tr>
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<td>17</td>
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<td>M</td>
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<td>35</td>
<td>M</td>
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<td>15</td>
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<td>M</td>
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<td>23</td>
<td>28</td>
<td>M</td>
</tr>
<tr>
<td>7</td>
<td>11</td>
<td>14</td>
<td>27</td>
<td>M</td>
</tr>
<tr>
<td>8</td>
<td>14</td>
<td>18</td>
<td>28</td>
<td>M</td>
</tr>
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<td>9</td>
<td>15</td>
<td>12</td>
<td>30</td>
<td>M</td>
</tr>
<tr>
<td>10</td>
<td>16</td>
<td>11</td>
<td>30</td>
<td>M</td>
</tr>
<tr>
<td>11</td>
<td>25</td>
<td>20</td>
<td>26</td>
<td>B</td>
</tr>
<tr>
<td>12</td>
<td>40</td>
<td>16</td>
<td>40</td>
<td>B</td>
</tr>
<tr>
<td>13</td>
<td>30</td>
<td>17</td>
<td>28</td>
<td>B</td>
</tr>
<tr>
<td>14</td>
<td>17</td>
<td>18</td>
<td>38</td>
<td>B</td>
</tr>
<tr>
<td>15</td>
<td>16</td>
<td>15</td>
<td>38</td>
<td>B</td>
</tr>
<tr>
<td>16</td>
<td>45</td>
<td>16</td>
<td>30</td>
<td>B</td>
</tr>
<tr>
<td>17</td>
<td>49</td>
<td>19</td>
<td>26</td>
<td>B</td>
</tr>
<tr>
<td>18</td>
<td>33</td>
<td>14</td>
<td>38</td>
<td>B</td>
</tr>
<tr>
<td>19</td>
<td>30</td>
<td>15</td>
<td>45</td>
<td>B</td>
</tr>
<tr>
<td>20</td>
<td>20</td>
<td>24</td>
<td>38</td>
<td>B</td>
</tr>
</tbody>
</table>

```r
pos <- bolt[,1]  
cw <- bolt[,2]  
ht <- bolt[,3]  
mb <- bolt[,4]  
ty <- factor(rep(mb, 3))  
is.factor(ty)  
plat <- factor(c(rep(1, 20), rep(2, 20), rep(3, 20)), labels=c("ht" , "po" , "cw"))  
y <- c(ht, po, cw)  
interaction.plot(ty, plat, y)  
options(contrasts = c("contr.treatment", "contr.poly"))  
#options(contrasts = c("contr.sum", "contr.poly"))  
fit <- lm(y ~ ty * plat)  
> anova(fit)  
```

Analysis of Variance Table

Response: y
Design-ho2.txt Tue Nov 04 11:26:56 2008

<table>
<thead>
<tr>
<th></th>
<th>Df</th>
<th>Sum Sq</th>
<th>Mean Sq</th>
<th>F value</th>
<th>Pr(&gt;F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ty</td>
<td>1</td>
<td>821.40</td>
<td>821.40</td>
<td>17.422</td>
<td>0.0001054 ***</td>
</tr>
<tr>
<td>plat</td>
<td>2</td>
<td>2290.63</td>
<td>1145.32</td>
<td>24.292</td>
<td>2.537e-08 ***</td>
</tr>
<tr>
<td>ty:plat</td>
<td>2</td>
<td>665.10</td>
<td>332.55</td>
<td>9.0916</td>
<td>0.0003952 ***</td>
</tr>
<tr>
<td>Residuals</td>
<td>56</td>
<td>2640.30</td>
<td>47.15</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

fit2<-lm(y ~ ty + plat)

anova(fit2)

Analysis of Variance Table

Response: y

<table>
<thead>
<tr>
<th></th>
<th>Df</th>
<th>Sum Sq</th>
<th>Mean Sq</th>
<th>F value</th>
<th>Pr(&gt;F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ty</td>
<td>1</td>
<td>821.40</td>
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<td>0.0001054 ***</td>
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<tr>
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<td>2</td>
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<td>2.537e-08 ***</td>
</tr>
<tr>
<td>Residuals</td>
<td>56</td>
<td>2640.30</td>
<td>47.15</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

>summary(fit)

Residuals:

<table>
<thead>
<tr>
<th></th>
<th>Min</th>
<th>1Q</th>
<th>Median</th>
<th>3Q</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-14.500</td>
<td>-2.525</td>
<td>0.100</td>
<td>2.600</td>
<td>18.500</td>
</tr>
</tbody>
</table>

Coefficients:

|       | Estimate | Std. Error | t value | Pr(>|t|) |
|-------|----------|------------|---------|---------|
| (Intercept) | 34.700 | 1.913 | 18.143 | < 2e-16 *** |
| tyM       | -5.300  | 2.705  | -1.960 | 0.05522 |
| platpo    | -4.200  | 2.705  | -1.553 | 0.12630 |
| platcw    | -17.300 | 2.705  | -6.396 | 3.92e-08 *** |
| tyM:platpo | -11.100 | 3.825  | -2.902 | 0.00536 ** |
| tyM:platcw | 4.800   | 3.825  | 1.255  | 0.21493 |

---

Signif. codes:  0 ’***’ 0.001 ’**’ 0.01 ’*’ 0.05 ’.’ 0.1 ’ ’ 1

Residual standard error: 6.048 on 54 degrees of freedom
Multiple R-Squared: 0.6566,   Adjusted R-squared: 0.6248
F-statistic: 20.65 on 5 and 54 DF,  p-value: 1.806e-11

Because tyM is significant so there is a significant
difference between bolt and mandrel. The significant
platCw signifies that there is a difference between (CW) and (HT).
Combined with the fact that there is no difference
between PO and HT it implies that there is a
difference between CW and PO.

The significant interaction term tyM:platpo
implies that the difference between PO and HT
varies from bolt to mandrel.

# Covariance Analysis}}

#Consider an experiment to study the breaking strength of a starch
#layer. The breaking strength is known to depend on the thickness of
#the layer. Because the thickness varies from run to run and its values
#cannot be controlled or chosen prior to the experiment, it should be
#treated as a covariate whose effect must be accounted for before
#comparing the three types of starch. The three types of starch are CA
#(canna), CO (corn), and PO (potato).
data<-read.table("LinMod-1/Design/starch.dat")
st<-factor(data[,1])
y<-data[,2]
x<-data[,3]-mean(data[,3])
options(contrasts=c("contr.helmert","contr.polynomial"))

fit<-lm(y˜x+st)
summary(fit)

#There is no difference between the three starch types. If we ignore
#the thickness we conclude (incorrectly) that there are differences
#between the three levels of starch.

> anova(lm(y˜st))

    Df Sum of Sq Mean Sq F Value        Pr(F)
    st  2   2246204 1123102 32.6191 1.512131e-09
    Residuals 46   1583818   34431

> plot(c(-5,5),c(300,1700),xlab="thick",ylab="y" ,type="n")
x1<-x[st==c("CA")]
y1<-y[st==c("CA")]
x2<-x[st==c("CO")]
y2<-y[st==c("CO")]
x3<-x[st==c("PO")]
y3<-y[st==c("PO")]
points(x1,y1)
points(x2,y2,pch=2)
points(x3,y3,pch=17)

fit1<-lm(y˜x)
fit1

legend(locator(1),c("CA","CO", "PO"), pch=c(8, 2, 17) )
abline(736.46,83.1336,lty=3)

z1<-rep(0,length(y))
z2<-rep(0,length(y))
z3<-rep(0,length(y))
z1[st==c("CA")]<-x[st==c("CA")]
z2[st==c("CO")]<-x[st==c("CO")]
z3[st==c("PO")]<-x[st==c("PO")]

options(contrasts=c("contr.treatment","contr.polynomial"))

fit2<-lm(y˜z1+z2+z3+st)
summary(fit2)

Coefficients:

            Estimate Std. Error t value Pr(>|t|)
(Intercept)   747.59      46.61 16.0400  <2e-16 ***
z1             59.29      22.65   2.6170   0.0122 *
z2            188.91      49.20   3.8390   0.0004 ***
z3             32.51      25.54   1.2730   0.2100
stCO          274.79     152.25   1.8050   0.0781 .
stPO          145.06      88.95   1.6310   0.1102
plot(c(-5,5),c(300,1700),xlab="thick",ylab="y",type="n")

x1<-x[st==c("CA")]
y1<-y[st==c("CA")]

x2<-x[st==c("CO")]
y2<-y[st==c("CO")]

x3<-x[st==c("PO")]
y3<-y[st==c("PO")]

points(x1,y1,pch=8)
points(x2,y2,pch=2)
points(x3,y3,pch=17)

legend(locator(1),c("CA","CO","PO"),pch=c(8,2,17))

abline(747.59+274.79,188.9,lty=3)
abline(747.59+145.06,32.5,lty=8)
#abline(600-10*32.51,32.51,lty=1)
legend(locator(1),c("CO","PO"),lty=c(3,8))

### Incomplete Block Designs 

### Simulated Example

<table>
<thead>
<tr>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>D</td>
<td>C</td>
<td>B</td>
</tr>
<tr>
<td>B</td>
<td>A</td>
<td>D</td>
<td>C</td>
</tr>
<tr>
<td>C</td>
<td>B</td>
<td>A</td>
<td>D</td>
</tr>
</tbody>
</table>

options(contrasts=c("contr.helmert","contr.poly"))

eff<-c(1,5,9,13)
y1<-rnorm(3)+10+10+c(1,5,9)
y2<-rnorm(3)+20+c(13,1,5)
y3<-rnorm(3)+30+c(9,13,1)
y4<-rnorm(3)+40+c(5,9,13)

#r=3,k=3,lambda=2,t=4,b=4

y<-c(y1,y2,y3,y4)
ttt<-factor(c(1,2,3,4,1,2,3,4,1,2,3,4),labels=c("A","B","C","D"))
blk<-factor(rep(1:4,rep(3,4)),labels=c("I","II","III","IV"))

fit1<-lm(y˜blk+ttt)
summary(aov(fit1))

fit1$coef

(Intercept) blk1 blk2 blk3 ttt1 ttt2 ttt3
 32.52299  -15.28868  -4.792636  6.078951  -5.80946  -2.289346  2.202005

tauhat<-dummy.coef(fit1)$ttt

tauhat
A second independent analysis can be made in which the block effects are regarded as random, and the block totals regarded as a second derived response variable.

I       II      III     IV
============================
A       D       C       B
B       A       D       C
C       B       A       D
============================

n1<-c(1,1,1,0)
n2<-c(1,1,0,1)
n3<-c(1,0,1,1)
n4<-c(0,1,1,1)
N<-rbind(n1,n2,n3,n4)

#B contains the block totals
B<-tapply(y,ttt,sum)
tautilde<-(N%*%B-3*3*mean(y))/(3-2)
tautilde<-as.vector(tauhat)

> tautilde
A        B              C       D

fit2<-lm(y˜ttt+blk)
summary(aov(fit2))

sigmasq<-1.57
sigmaBsq<-(419.29-1.57)*3/(4*2)

vartau1<-(sigmasq*3*3/(2*4*4)
vartau2<-(sigmasq+3*sigmaBsq)*3*3/4*1
wei1<-1/sqrt(vartau1)
wei2<-1/sqrt(vartau2)
w<-wei1/(wei1+wei2)

taufinal<-(w*tauhat+(1-w)*tautilde)

> taufinal
A         B         C         D
-6.715241 -1.850736  2.807592  5.758386
# you can redo the analysis assuming that there are small block effects
# (blocking is inefficient) and see the change in the weights.

options(contrasts=c("contr.helmert","contr.poly"))

#TREATMENT EFFECTS ARE:
eff<-c(1,5,9,13)
y1<-rnorm(3)+c(1,5,9)
y2<-rnorm(3)-0.1+c(13,1,5)
y3<-rnorm(3)+c(9,13,1)
y4<-rnorm(3)+0.1+c(5,9,13)

#r=3,k=3,lambda=2,t=4,b=4
y<-c(y1,y2,y3,y4)
ttt<-factor(c(1,2,3,4,1,2,3,4,1,2,3,4),labels=c("A","B","C","D"))
blk<-factor(rep(1:4,rep(3,4)),labels=c("I","II","III","IV"))

fit1<-lm(y˜blk+ttt)
summary(aov(fit1))

Df  Sum Sq Mean Sq F value    Pr(>F)
blk          3  32.205  10.735  7.6511 0.0257384 *
ttt          3 162.171  54.057 38.5282 0.0007026 ***
Residuals    5   7.015   1.403

tauhat<-dummy.coef(fit1)$ttt
tauhat
A         B         C         D
-5.703227 -1.108250  2.016338  4.795139

n1<-c(1,1,1,0)
n2<-c(1,1,0,1)
n3<-c(1,0,1,1)
n4<-c(0,1,1,1)
N<-rbind(n1,n2,n3,n4)

#B contains the block totals
B<-tapply(y,ttt,sum)
tauprimitive<-(N%*%B-3*3*mean(y))/(3-2)
tauprimitive<-as.vector(tauhat)
tauprimitive


fit2<-lm(y˜ttt+blk)
summary(aov(fit2))

Df  Sum Sq Mean Sq F value    Pr(>F)
ttt          3 188.842  62.947 44.8648 0.0004883 ***
blk          3   5.533   1.844  1.3146 0.3672256
Residuals    5   7.015   1.403
Design-ho2.txt Tue Nov 04 11:26:56 2008 7

\text{sigmaBsq} <- (1.844 - 1.403)*3/(4*2)

\text{vartau2} <- (\text{sigmasq} + 3*\text{sigmaBsq})*3/4*1
\text{wei2} <- 1/\sqrt{\text{vartau2}}
\text{w} <- \text{wei1}/(\text{wei1} + \text{wei2})
\text{taufinal} <- \text{w} * \tau_{\text{hat}} + (1 - \text{w}) * \tau_{\text{tilde}}

A         B         C         D
-7.753988 -2.545277  2.557203  7.742062

# Tire Example

Consider the following experiment made to assess the effect of rubber compounds in a tire on the lifetime of the tires. Because of manufacturing limitations a tire can be divided only into at most three sections; therefore only three compounds can be compared at one time. As a particular tire is tested the three parts are all subjected to the same road conditions. The design used was a balanced incomplete block design as there were 4 types of compounds that required testing.

| Tire | Compound | A     | B     | C | D \\
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>238</td>
<td>238</td>
<td>279</td>
<td>*</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>196</td>
<td>213</td>
<td>*</td>
<td>308</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>254</td>
<td>*</td>
<td>334</td>
<td>367</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>*</td>
<td>312</td>
<td>421</td>
<td>412</td>
</tr>
</tbody>
</table>

data <- read.table("LinMod-1/Design/tire.dat")
tire <- factor(data[,1], labels=c("1","2","3","4"))
comp <- factor(data[,2], labels=c("A","B","C","D"))
y <- data[,3]

# tire is the block
# comp is the treatment

options(contrasts=c("contr.helmert","contr.poly"))

fit.intra <- lm(y ~ tire + comp)
anova(fit.intra)
tauhat <- dummy.coef(fit.intra)$comp
tauhat

A       B       C       D
-45.375 -41.000  30.875  55.500

Analysis of Variance Table

<table>
<thead>
<tr>
<th>Response: y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Df</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>tire</td>
</tr>
</tbody>
</table>
A second analysis can be made in which the block effects are regarded as random, and the block totals regarded as a second derived response variable.

### Data Table

<table>
<thead>
<tr>
<th>Tire\Compound</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>238</td>
<td>238</td>
<td>279</td>
<td>*</td>
</tr>
<tr>
<td>2</td>
<td>196</td>
<td>213</td>
<td>*</td>
<td>308</td>
</tr>
<tr>
<td>3</td>
<td>254</td>
<td>*</td>
<td>334</td>
<td>367</td>
</tr>
<tr>
<td>4</td>
<td>*</td>
<td>312</td>
<td>421</td>
<td>412</td>
</tr>
</tbody>
</table>

### R Code

```r
n1 <- c(1, 1, 1, 0)
n2 <- c(1, 1, 0, 1)
n3 <- c(1, 0, 1, 1)
n4 <- c(0, 1, 1, 1)
N <- rbind(n1, n2, n3, n4)
B <- tapply(y, comp, sum)
taulilde <- (N %*% B - 3*3*mean(y))/(3-2)
taulilde <- as.vector(taulilde)
taulilde
```

```r
fit.inter <- lm(y ~ comp + tire)
anova(fit.inter)
```

### ANOVA Table

<table>
<thead>
<tr>
<th></th>
<th>Df</th>
<th>Sum Sq</th>
<th>Mean Sq</th>
<th>F value</th>
<th>Pr(&gt;F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>comp</td>
<td>3</td>
<td>38814</td>
<td>12938</td>
<td>36.946</td>
<td>0.0007763 ***</td>
</tr>
<tr>
<td>tire</td>
<td>3</td>
<td>21038</td>
<td>7013</td>
<td>20.026</td>
<td>0.0032406 **</td>
</tr>
<tr>
<td>Residuals</td>
<td>5</td>
<td>1751</td>
<td>350</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

```r
sigmasq <- 350
sigmaBsq <- (7013 - 350)*3/(4*2)
vartaul <- sigmasq*3/((2*4*4)
vartau2 <- (sigmasq + 3*sigmaBsq)*3/4*1
weil <- 1/sqrt(vartaul)
wei2 <- 1/sqrt(vartau2)
w <- weil/(weil + wei2)
taufinal <- w*tauhat + (1-w)*tautilde
taufinal
```

### Taum Final Values

```
A         B         C         D
-55.70222 -47.94851  37.76271  65.88802
```